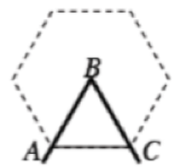
Agenda: #71F Ch8-1

Objectives:

- About special types of polygons, such as regular and non−convex polygons.
- How the measures of the interior and exterior angles of a regular polygon are related to the number of sides of the polygon.

1) Warm Up (10min.)

The angle created by a hinged mirror when forming a regular polygon is called a **central angle**. For example, $\angle ABC$ in the diagram at right is the central angle of the regular hexagon.



a) Find a central angle for regular hexagon.

b) Find an interior angle for regular hexagon.

c) Find an exterior angle for regular hexagon.

2) (10min)

3) (10min)

4) (10min)

5) (10min)

6) Closing Activities (10 min)

8.1.1 How can I build it?

Pinwheels and Polygons

In previous chapters, you have studied triangles and quadrilaterals. In Chapter 8, you will broaden your focus to include all polygons and will study what triangles can tell us about shapes with 5, 8, or even 100 sides.

By the end of this lesson, you should be able to answer these questions:

How can you use the number of sides of a regular polygon to find the measure of the central angle?

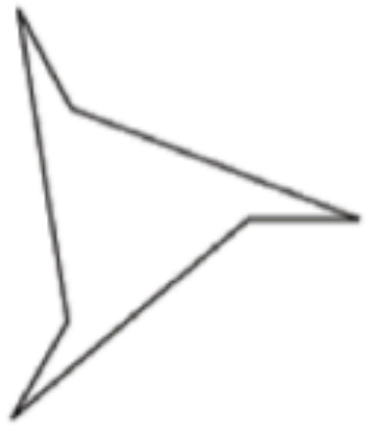
What type of triangle is needed to form a regular polygon?

8-1. PINWHEELS AND POLYGONS

Inez loves pinwheels. One day in class, she noticed that if she put three congruent triangles together so that one set of corresponding angles are adjacent, she could make a shape that looks like a pinwheel.

a. Can you determine any of the angles of her triangles? Explain how you found your answer.

b. The overall shape (outline) of Inez's pinwheel is shown at right. How many sides does it have? What is another name for this shape?



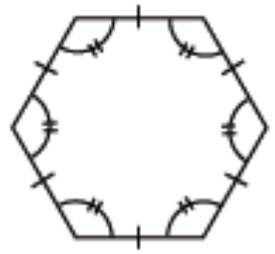
METHODS AND MEANINGS

MATH NOTES

Convex and Non-Convex Polygons

A **polygon** is defined as a two-dimensional closed figure made up of straight line segments connected end-to-end. These segments may not cross (intersect) at any other points.

A polygon is referred to as a **regular polygon** if it is equilateral (all sides have the same length) and equiangular (all interior angles have equal measure). For example, the hexagon shown at right is a regular hexagon because all sides have the same length and each interior angle has the same measure.



A polygon is called **convex** if each pair of interior points can be connected by a segment without leaving the interior of the polygon. See the example of convex and non-convex shapes in problem 8-4.

8.1.2 What is its measure?

Interior Angles of Polygons

In an earlier chapter you discovered that the sum of the interior angles of a triangle is always 180°. What about other polygons, such as hexagons or decagons? What about the sum of their interior angles? Do you think it matters if the polygon is convex or not? Consider these questions today as you investigate the angles of a polygon.

8-13. Copy the diagram of the regular pentagon at right onto your paper. Then, with your team, find the *sum* of the measures of its interior angles *as many ways as you can*. You may want to use the fact that the sum of the angles of a triangle is 180°. Be prepared to share your team's methods with the class.

8-14 a. Complete the table

Number of Sides of the Polygon	3	4	5	6	7	8	9	10	12
Sum of the Interior Angles of the Polygon	180°								

b. **Does the interior angle sum depend on whether the polygon is convex? Test this idea by drawing a few non-convex polygons (like the one at right) on your paper and determine if it matters whether the polygon is convex. Explain your findings.**

c. **Find the sum of the interior angles of a 100-gon. Explain your reasoning.**

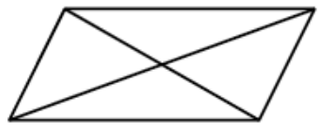
METHODS AND MEANINGS

MATH NOTES

Special Quadrilateral Properties

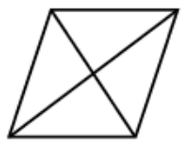
In Chapter 7, you examined several special quadrilaterals and proved conjectures regarding many of their special properties. Review what you learned below.

Parallelogram: Opposite sides of a parallelogram are congruent and parallel. Opposite angles are congruent. Also, since the diagonals create two pair of congruent triangles, the diagonals bisect each other.



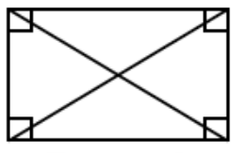
Parallelogram

Rhombus: Since a rhombus is a parallelogram, it has all of the properties of a parallelogram. In addition, its diagonals are perpendicular bisectors that bisect the angles of the rhombus; the diagonals also create four congruent triangles.



Rhombus

Rectangle: Since a rectangle is a parallelogram, it has all of the properties of a parallelogram. In addition, its diagonals must be congruent.



Rectangle

Isosceles Trapezoid: The base angles (angles joined by a base) of an isosceles trapezoid are congruent.

Isosceles Trapezoid

8.1.3 What if it is a regular polygon?

Angles of Regular Polygons

In Lesson 8.1.2 you discovered how to determine the sum of the interior angles of a polygon with any number of sides. What more can you learn about a polygon? Today you will focus on the interior and exterior angles of regular polygons.

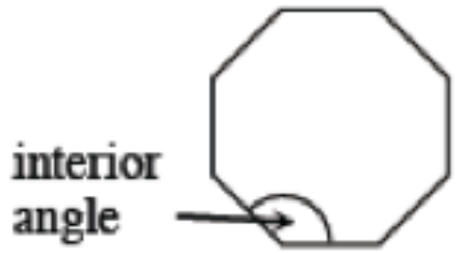
As you work today, keep the following focus questions in mind:

Does it matter if the polygon is regular?

Is there another way to find the answer?

What's the connection?

a. Determine the measure of each interior angle of a regular octagon. Explain how you found your answer.

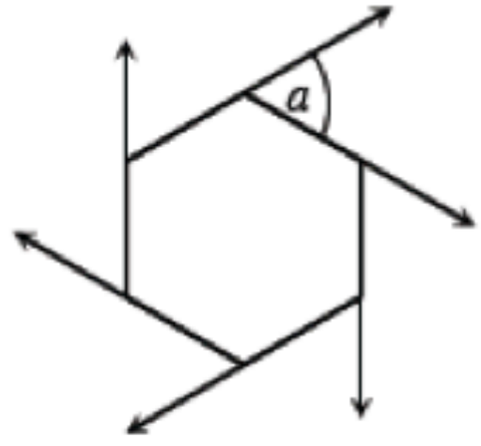


b. What about the interior angles of other regular polygons? Find the interior angles of a regular nonagon and a regular 100-gon.

c. Will the process you used for part (a) work for any regular polygon? Write an expression that will calculate the interior angle of an n-gon.

8-26. Jeremy asks, "*What about exterior angles? What can we learn about them?*"

a. Examine the regular hexagon shown at right. Angle a is an example of an **exterior angle** because it is formed on the outside of the hexagon by extending one of its sides. Are all of the exterior angles of a regular polygon equal? Explain how you know.



b. Find a. Be prepared to share how you found your answer.

c. This regular hexagon has six exterior angles, as shown in the diagram above. What is the sum of the exterior angles of a regular hexagon?

d. What can you determine about the exterior angles of other regular polygons? Explore this with your team. Have each team member choose a different shape from the list below to analyze. For each shape:

- Find the measure of one exterior angle of that shape, and
- Find the sum of the exterior angles.

(1) equilateral triangle

(2) regular octagon

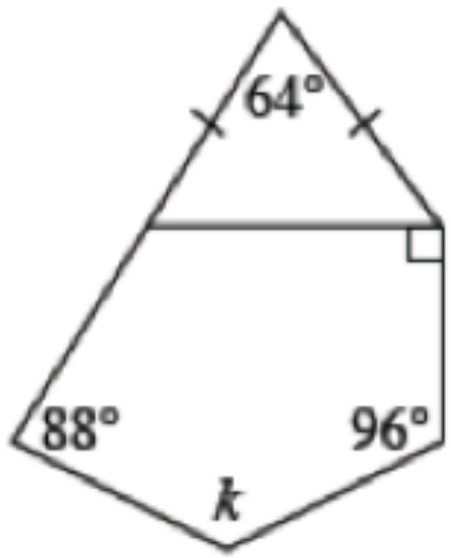
(3) regular decagon

(4) regular dodecagon (12-gon)

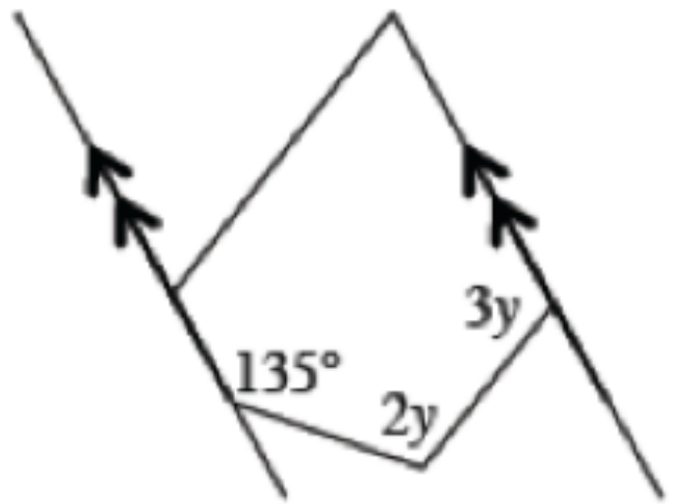
Homework Section

Use the angle relationships in each of the diagrams below to solve for the given variables. Show all work.

c.



d.



Name of Polygon	Number of Sides
Triangle	3
[illegible]	4
Pentagon	5
[illegible]	6
[illegible]	7

Name of Polygon	Number of Sides
Octagon	8
[illegible]	9
[illegible]	10
11-gon	11
n-gon	*n*

8-17. On graph paper, graph ΔABC if $A(3, 0)$, $B(2, 7)$, and $C(6, 4)$.

a. What is the most specific name for this triangle? Prove your answer is correct using both slope and side length.

b. Find $m\angle A$. Explain how you found your answer.

8-18. The exterior angles of a quadrilateral are labeled a, b, c, and d in the diagram at right. Find the measures of a, b, c, and d and then find the sum of the exterior angles.