Agenda: #70F Ch7-3

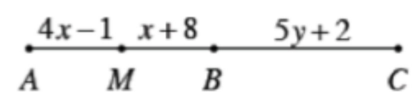
Objectives:

- How to find the midpoint of a line segment.
- How to use algebraic tools to explore quadrilaterals on coordinate axes.

1) Warm Up (10min.)

Preview

7-65. Point M is the midpoint of $\overline{AB}$ and B is the midpoint of $\overline{AC}$. What are the values of x and y? Show all work and reasoning.



2) 7-126 Graph Triangle ABC and Calculate midpoints(10min)

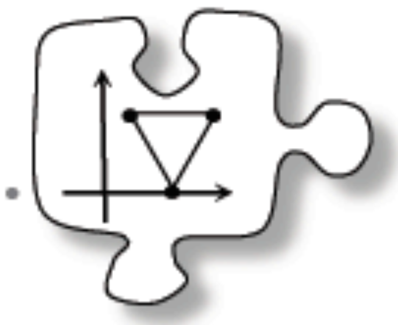
3) 7-128 Examine the triangle (10min)

4) Must be, Could be (10min)

5) Shape Factory (10min)

6) Closing Activities (10 min)

7.3.2 How can I find the midpoint?



Coordinate Geometry and Midpoints

In Lesson 7.3.1, you applied your existing algebraic tools to analyze geometric shapes on a coordinate grid. What other algebraic processes can help us analyze shapes? And what else can be learned about geometric shapes?

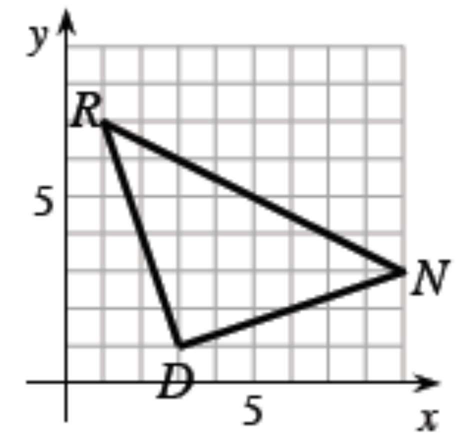
7-126. Cassie wants to confirm her theorem on triangle midsegments (from Lesson 7.2.6) using a coordinate grid. She started with ΔABC with $A(0,0)$, $B(2,6)$, and $C(7,0)$.

a. Graph ΔABC on graph paper.

b. With your team, find the coordinates of P, the midpoint of $\overline{AB}$. Likewise, find the coordinates of Q, the midpoint of $\overline{BC}$.

c. Prove that the length of the midsegment, $\overline{PQ}$, is half the length of $\overline{AC}$. Also verify that $\overline{PQ}$ is parallel to $\overline{AC}$.

7-128. Randy has decided to study the triangle graphed at right.



a. Consider all the special properties this triangle can have. Without using any algebra tools, predict the best name for this triangle.

b. For your answer to part (a) to be correct, what is the minimum amount of information that must be true about ΔRND?

c. Use your algebra tools to verify each of the properties you listed in part (b). If you need, you may change your prediction of the shape of ΔRND.

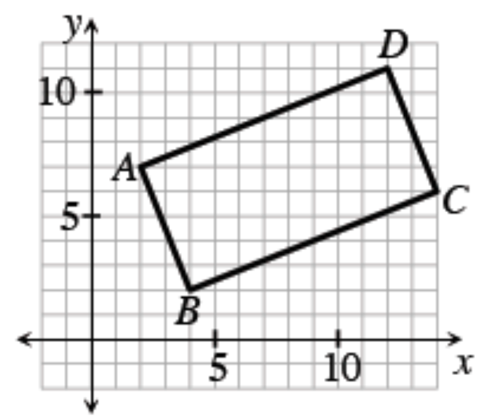
d. Randy wonders if there is anything special about the midpoint of $\overline{RN}$. Find the midpoint M, and then find the lengths of $\overline{RM}$, $\overline{DM}$, and $\overline{MN}$. What do you notice?

METHODS AND MEANINGS

MATH NOTES

Coordinate Geometry

Coordinate geometry is the study of geometry on a coordinate grid. Using common algebraic and geometric tools, you can learn more about a shape, such as, *"Does it have a right angle?"* or *"Are there two sides with the same length?"*



One useful tool is the Pythagorean Theorem. For example, the Pythagorean Theorem could be used to determine the length of side $\overline{AB}$ of $ABCD$ at right. By drawing the slope triangle between points A and B, the length of $\overline{AB}$ can be found to be $\sqrt{2^2+5^2}=\sqrt{29}$ units.

Similarly, slope can help analyze the relationships between the sides of a shape. If the slopes of two sides of a shape are equal, then those sides are **parallel**. For example, since the slope of $\overline{BC}=\frac{2}{5}$ and the slope of $\overline{AD}=\frac{2}{5}$, then $\overline{BC} \,//\, \overline{AD}$.

Also, if the slopes of two sides of a shape are opposite reciprocals, then the sides are **perpendicular** (meaning they form a 90° angle). For example, since the slope of $\overline{BC}=\frac{2}{5}$ and the slope of $\overline{AB}=-\frac{5}{2}$, then $\overline{BC}\perp\overline{AB}$.

By using multiple algebraic and geometric tools, you can identify shapes. For example, further analysis of the sides and angles of $ABCD$ above shows that $AB = DC$ and $BC = AD$. Furthermore, all four angles measure 90°. These facts together indicate that $ABCD$ must be a rectangle.

7.3.3 What kind of quadrilateral is it?

Identifying Quadrilaterals on a Coordinate Grid

Today you will use algebra tools to investigate the properties of a quadrilateral and then will use those properties to identify the type of quadrilateral it is.

7-138. MUST BE, COULD BE

Mr. Quincey has some new challenges for you! For each description below, decide what special type the quadrilateral *must be* and/or what special type the quadrilateral *could be*. Look out: Some descriptions may have no *must be* statements, and some descriptions may have many *could be* statements!

a. My quadrilateral has three right angles.

b. My quadrilateral has a pair of parallel sides.

c. My quadrilateral has two consecutive equal angles.

7-139. THE SHAPE FACTORY

You just got a job in the Quadrilaterals Division of your uncle's Shape Factory. In the old days, customers called up your uncle and described the quadrilaterals they wanted over the phone: "*I'd like a parallelogram with…*".

"*But nowadays,*" your uncle says, "*customers using computers have been emailing orders in lots of different ways*." Your uncle needs your team to help analyze his most recent orders listed below to identify the quadrilaterals and help the shape-makers know what to produce.

Your Task: For each of the quadrilateral orders listed below,

- Create a diagram of the quadrilateral on graph paper.
- Decide if the quadrilateral ordered has a special name. To help the shape-makers, your name must be as specific as possible. (For example, do not just call a shape a rectangle when it is also a square!)
- Record and be ready to present a proof that the quadrilateral ordered must be the kind you say it is. It is not enough to say that a quadrilateral *looks* like it is of a certain type or *looks* like it has a certain property. Customers will want to be sure they get the type of quadrilateral they ordered!

Discussion Points

What special properties might a quadrilateral have?

What algebra tools could be useful?

What types of quadrilaterals might be ordered?

The orders:

a. A quadrilateral formed by the intersection of these lines:

$y=-\frac{3}{2}x+3$ $\quad$ $y=\frac{3}{2}x-3$ $\quad$ $y=-\frac{3}{2}x+9$ $\quad$ $y=\frac{3}{2}x+3$

b. A quadrilateral with vertices at these points:

$A(0,2)$ $\quad$ $B(1,0)$ $\quad$ $C(7,3)$ $\quad$ $D(4,4)$

c. A quadrilateral with vertices at these points:

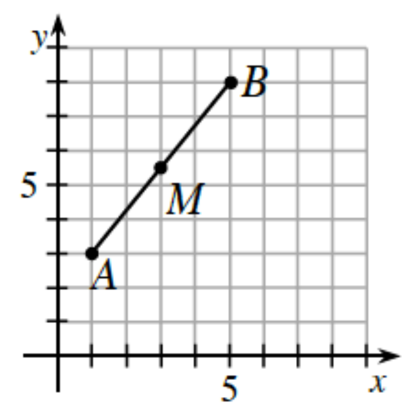
$W(0,5)$ $\quad$ $X(2,7)$ $\quad$ $Y(5,7)$ $\quad$ $Z(5,1)$

METHODS AND MEANINGS

MATH NOTES

Finding a Midpoint

A **midpoint** is a point that divides a line segment into two parts of equal length. For example, M is the midpoint of $\overline{AB}$ at right.



There are several ways to find the midpoint of a line segment if the coordinates of the endpoints are known. One way is to add half the change in x ($\frac{1}{2}\Delta x$) and half of the change in y ($\frac{1}{2}\Delta y$) to the x- and y-coordinates of the starting point, respectively.

Thus, if $A(1,3)$ and $B(5,8)$, then $\Delta x = 5-1=4$ and $\Delta y = 8-3=5$. Then the x-coordinate of M is $1+\frac{1}{2}(4)=3$ and the y-coordinate is $3+\frac{1}{2}(5)=5.5$. So point M is at $(3,5.5)$.

This strategy can be used to find other points between A and B that are a proportion of the way from a starting point. For example, if you wanted to find a point $\frac{4}{5}$ of the way from point A to point B, then this could be found by adding $\frac{4}{5}$ of Δx to the x-coordinate of point A and adding $\frac{4}{5}$ of Δy to the y-coordinate of point A. This would be the point $((1+\frac{4}{5}(4), 3+\frac{4}{5}(5))$ which is $(4.2,7)$. Generally, a point a ratio r from $A(x_0, y_0)$ to $B(x_1, y_1)$ is at $(x_0 + r(x_1 - x_0), y_0 + r(y_1 - y_0))$.

Homework from this page.

7-131. Tomika remembers that the diagonals of a rhombus are perpendicular to each other.

a. Graph $ABCD$ if $A(1, 4)$, $B(6, 6)$, $C(4, 1)$, and $D(-1, -1)$. Is $ABCD$ a rhombus? Show how you know.

b. Find the equation of the lines on which the diagonals lie. That is, find the equations of $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$.

c. Compare the slopes of $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$. What do you notice?

7-135. Consider ΔABC with vertices $A(2, 3)$, $B(6, 6)$, and $C(8, -5)$.

a. Draw ΔABC on graph paper. What kind of triangle is ΔABC? Prove your result.

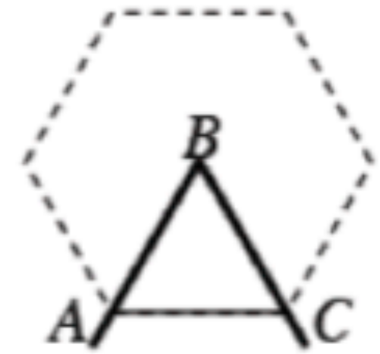
b. Reflect ΔABC across $\overline{AC}$. Find the location of B'. What name best describes the resulting figure? Prove your claim.

7-140. Each problem below gives the endpoints of a segment. Find the coordinates of the midpoint of the segment. If you need help, consult the Math Notes box for this lesson.

a. (5, 2) and (11, 14)

b. (3, 8) and (10, 4)

7-144. The angle created by a hinged mirror when forming a regular polygon is called a **central angle**. For example, $\angle ABC$ in the diagram at right is the central angle of the regular hexagon.



a. If the central angle of a regular polygon measures 18°, how many sides does the polygon have?

b. Can a central angle measure 90°? 180°? 13°? For each angle measure, explain how you know.