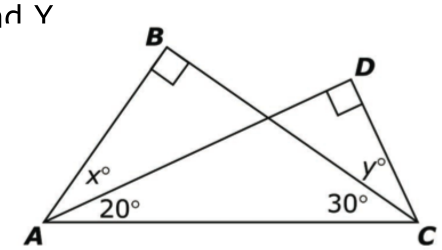
Agenda: #69F Ch7-2

1) Objectives

- How to use algebraic tools to explore quadrilaterals on coordinate axes.

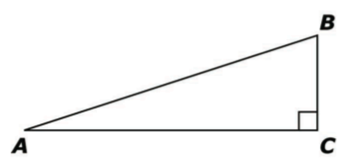
Warm Up (15min.) SBAC Released Q.

Right triangle *ABC* and right triangle *ACD* overlap as shown below. Find x and Y



c) Advanced students only

In right triangle *ABC*, side *AC* is longer than side *BC*. The boxed numbers represent the possible side lengths of triangle *ABC*.



7	8
15	17
18	20
24	25

not drawn to scale

Identify three boxed numbers that could be the side lengths of triangle *ABC*. Enter the number you chose to represent the length of each side.

2) Graph Inequality Question (15min)

3) Parallelograms and Triangles (15min)

4) Kite Questions (15min)

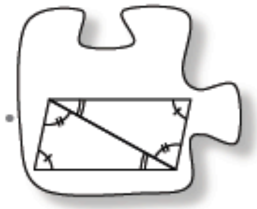
5) Conjecture Questions (15min)

6) Closing Activities (15 min)

7-71. On graph paper, graph and shade the solutions for the inequality below.

$$y < -\frac{2}{3}x + 5$$

7.2.1 What can congruent triangles tell me?



Special Quadrilaterals and Proof

In earlier chapters you studied the relationships between the sides and angles of a triangle, and solved problems involving congruent and similar triangles. Now you are going to expand your study of shapes to quadrilaterals. What can triangles tell you about parallelograms and other special quadrilaterals?

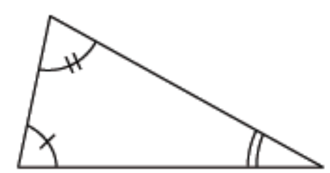
By the end of this lesson, you should be able to answer these questions:

What are the relationships between the sides, angles, and diagonals of a parallelogram?

How are congruent triangles useful?

7-49. Carla is thinking about parallelograms and wondering if there are as many special properties for parallelograms as there are for triangles. She remembers that it is possible to create a shape that looks like a parallelogram by rotating a triangle about the midpoint of one of its sides.

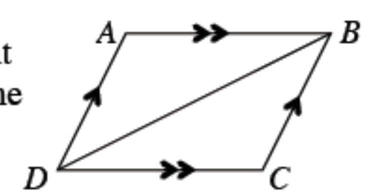
a. Carefully trace the triangle at right onto tracing paper. Be sure to copy the angle markings as well. Then rotate the triangle about a midpoint of a side to make a shape that looks like a parallelogram.



b. Is Carla's shape truly a parallelogram? Use the angles to convince your teammates that the opposites sides must be parallel. Then write a convincing argument.

c. What else can the congruent triangles tell you about a parallelogram? Look for any relationships you can find between the angles and sides of a parallelogram.

d. Does this work for all parallelograms? That is, does the diagonal of a parallelogram always split the shape into two congruent triangles? Draw the parallelogram at right on your paper. Knowing only that the opposite sides of a parallelogram are parallel, create a flowchart to show that the triangles are congruent.



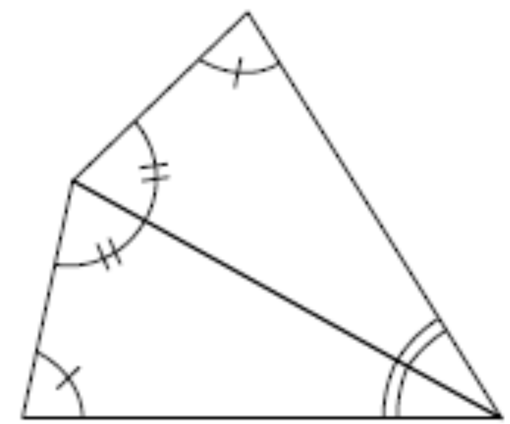
7-50. CHANGING A FLOWCHART INTO A PROOF

The flowchart you created for part (d) of problem 7-49 shows how you can conclude that if a quadrilateral is a parallelogram, then its each of its diagonals splits the quadrilateral into two congruent triangles.

However, to be convincing, the facts that you listed in your flowchart need to have justifications. This shows the reader how you know the facts are true and helps to prove your conclusion.

Therefore, with the class or your team, decide how to add reasons to each statement (bubble) in your flowchart. You may need to add more bubbles to your flowchart to add justification and to make your proof more convincing.

7-51. Kip is confused. He put his two triangles from problem 7-49 together as shown at right, but he did not get a parallelogram.

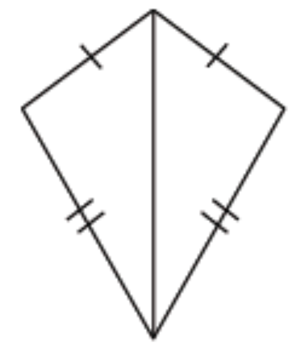


a. What shape did he make? Justify your conclusion.

b. What transformation(s) did Kip use to form his shape?

c. What do the congruent triangles tell you about the angles of this shape?

7-52. KITES

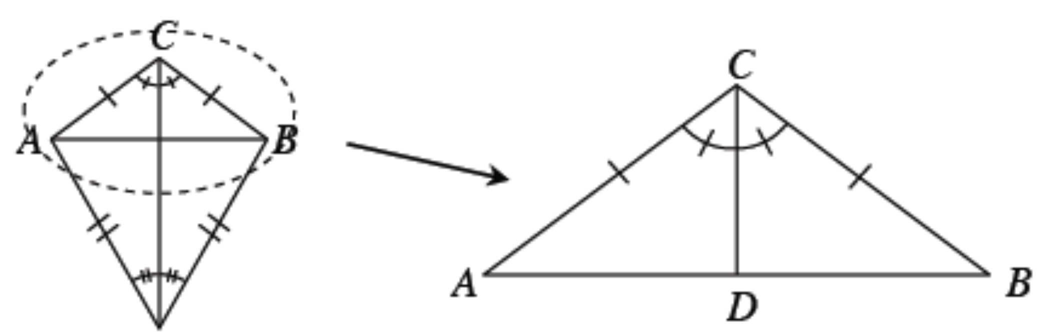


Kip shared his findings about his kite with his teammate, Carla, who wants to learn more about the diagonals of a kite. Carla quickly sketched the kite at right onto her paper with a diagonal showing the two congruent triangles.

a. **EXPLORE:** Trace this diagram onto tracing paper and carefully add the other diagonal. Then, with your team, consider how the diagonals may be related. Use tracing paper to help you explore the relationships between the diagonals. If you make an observation you think is true, move on to part (b) and write a conjecture.

b. **CONJECTURE:** If you have not already done so, write a conjecture based on your observations in part (a).

c. **PROVE:** When she drew the second diagonal, Carla noticed that four new triangles appeared. "*If any of these triangles are congruent, then they may be able to help us prove our conjecture from part (b),*" she said. Examine ΔABC below. Are ΔACD and ΔBCD congruent? Create a flowchart proof like the one from problem 7-50 to justify your conclusion.



d. Now extend your proof from part (c) to prove your conjecture from part (b).

7-53. Reflect on all of the interesting facts about parallelograms and kites you have proven during this lesson. Obtain a Theorem Toolkit (Lesson 7.2.1A Resource Page) from your teacher. On it, record each **theorem** (proven conjecture) that you have proven about the sides, angles, and diagonals of a parallelogram in this lesson. Do the same for a kite. Be sure your diagrams contain appropriate markings to represent equal parts.

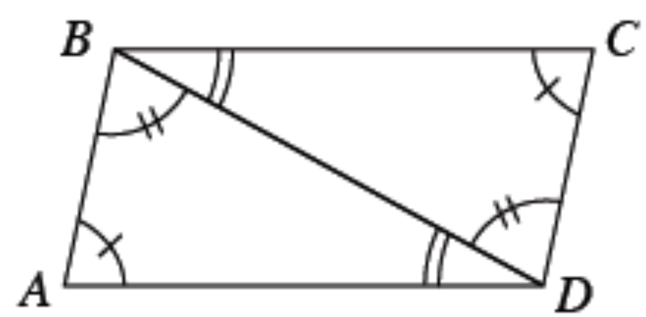
Methods and Meanings

MATH NOTES

Reflexive Property of Equality

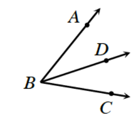
In this lesson, you used the fact that two triangles formed by the diagonal of a parallelogram share a side of the same length to help show that the triangles were congruent.

The **Reflexive Property of Equality** states that the measure of any side or angle is equal to itself. For example, in the parallelogram at right, $\overline{BD} \cong \overline{DB}$ because of the Reflexive Property.

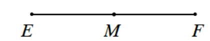


Review & Preview

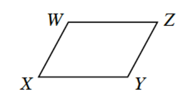
7-54. Use the information given for each diagram below to solve for x. Show all work.



a. $\overline{BD}$ bisects $\angle ABC$. (Remember that this means it divides the angle into two equal parts.) If $m\angle ABD = 5x - 10°$ and $m\angle ABC = 65°$, solve for x.



b. Point M is a midpoint of $\overline{EF}$. If $EM = 4x - 2$ and $MF = 3x + 9$, solve for x.



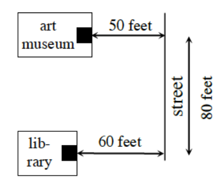
c. $WXYZ$ at right is a parallelogram. If $m\angle W = 9x - 3°$ and $m\angle Z = 3x + 15°$, solve for x.

7-55. Jamal used a hinged mirror to create a regular polygon like you did in Lesson 7.1.4.

a. If his hinged mirror formed a 72° angle and the core region in front of the mirror was isosceles, how many sides did his polygon have?

b. Now Jamal has decided to create a regular polygon with 9 sides, called a nonagon. If his core region is again isosceles, what angle is formed by his mirror?

7-56. Sandra wants to park her car so that she optimizes the distance she has to walk to the art museum and the library. That is, she wants to park so that her total distance directly to each building is the shortest. Find where she should park.



7-57. Write an equation for each of the following sequences.

a. $40, 60, 80, \ldots$

b. $3, \frac{3}{2}, \frac{3}{4}, \ldots$

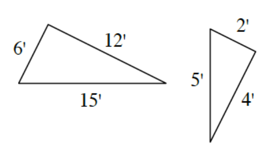
7-58. Earl (from Chapter 6) still hates to wash the dishes and take out the garbage. He found his own weighted coin, one that would randomly land on heads 30% of the time. He will flip a coin once for each chore and will perform the chore if the coin lands on heads.

a. What is the probability that Earl will get out of doing both chores?

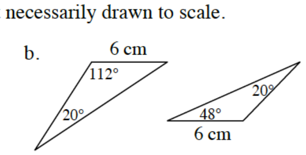
b. What is the probability that Earl will have to take out the garbage, but will not need to wash the dishes?

7-59. Which pairs of triangles below are congruent and/or similar? For each part, explain how you know using an appropriate triangle congruence or similarity condition. Note: The diagrams are not necessarily drawn to scale.

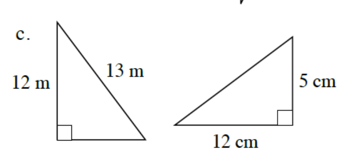
a.



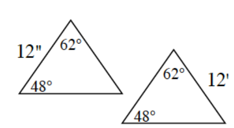
b.



c.



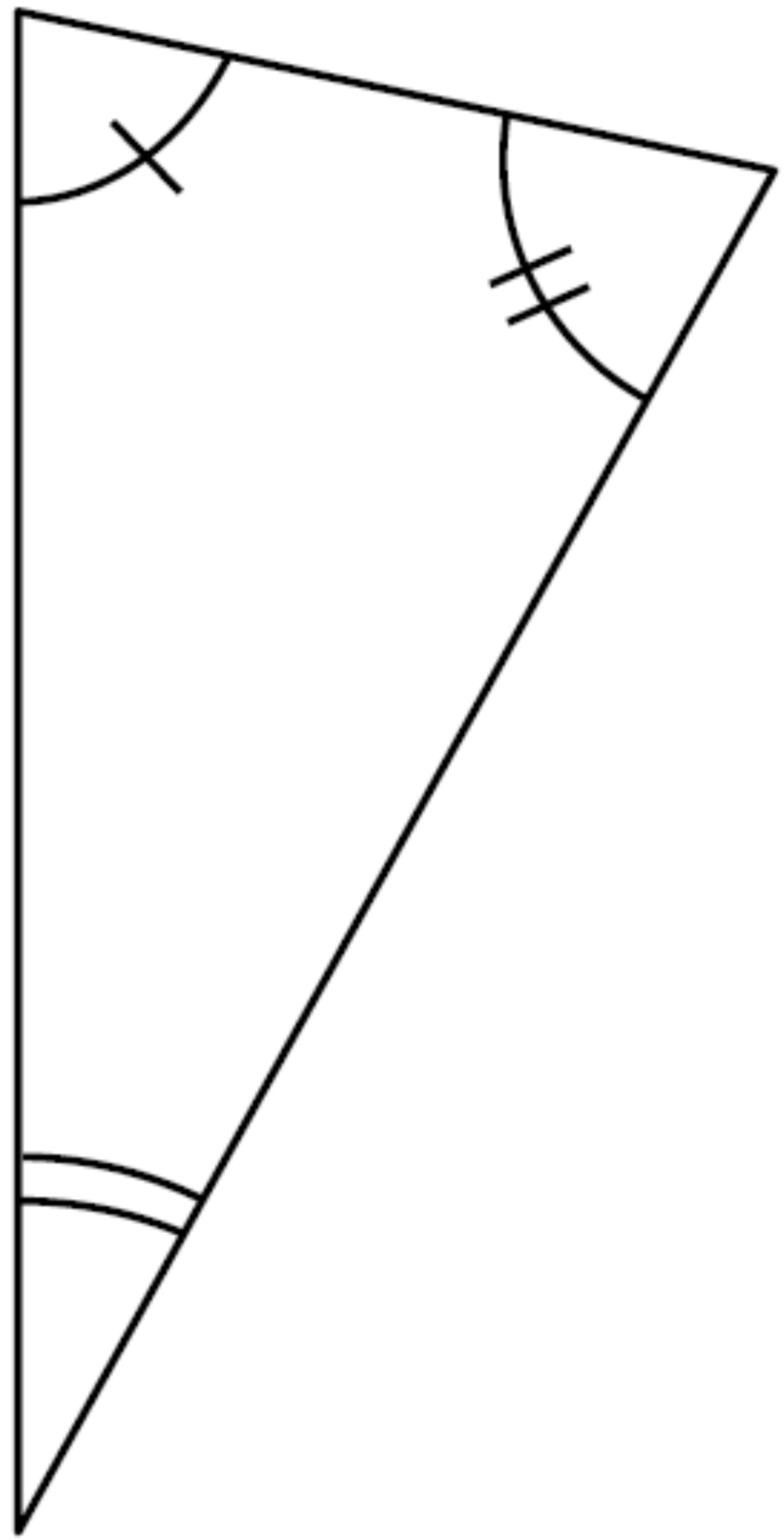
d.



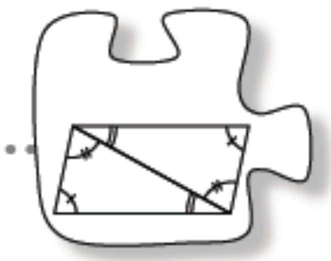
7-60. For part (b) of problem 7-59, explain how the triangles are congruent using a sequence of rigid transformations.

Theorem Toolkit

Parallelograms 	Kites 



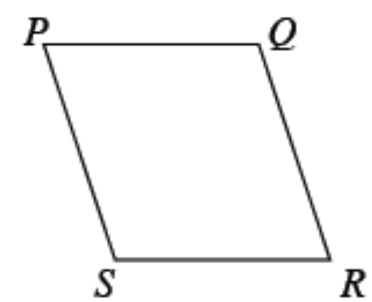
7.2.2 What is special about a rhombus?



Properties of Rhombi

In Lesson 7.2.1, you learned that congruent triangles can be a useful tool to discover new information about parallelograms and kites. But what about other quadrilaterals? Today you will use congruent triangles to investigate and prove special properties of rhombi (the plural of rhombus). At the same time, you will continue to develop your ability to make conjectures and prove them convincingly.

7-61. Audrey has a favorite quadrilateral – the rhombus. Even though a rhombus is defined as having four congruent sides, she suspects that the sides of a rhombus have other special properties.



a. **EXPLORE:** Draw a rhombus like the one at right on your paper. Mark the side lengths equal.

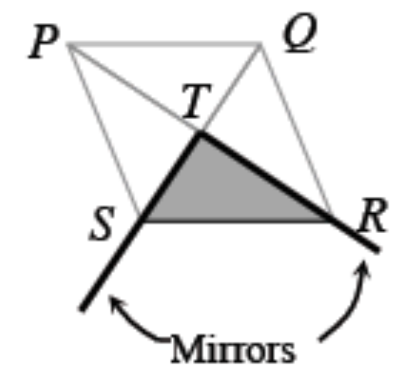
b. **CONJECTURE:** What else might be special about the sides of a rhombus? Write a conjecture.

c. **PROVE:** Audrey knows congruent triangles can help prove other properties about quadrilaterals. She starts by adding a diagonal $\overline{PR}$ to her diagram so that two triangles are formed. Add this diagonal to your diagram and prove that the created triangles are congruent. Then use a flowchart with reasons to show your logic. Be prepared to share your flowchart with the class.

d. How can the triangles from part (c) help you prove your conjecture from part (b) above? Discuss with the class how to extend your flowchart to convince others. Be sure to justify any new statements with reasons.

7-62. Now that you know the opposite sides of a rhombus are parallel, what else can you prove about a rhombus? Consider this as you answer the questions below.

a. **EXPLORE:** Remember that in Lesson 7.1.4, you explored the shapes that could be formed with a hinged mirror. During this activity, you used symmetry to form a rhombus. Think about what you know about the reflected triangles in the diagram. What do you think is true about the diagonals $\overline{SQ}$ and $\overline{PR}$? What is special about $\overline{ST}$ and $\overline{QT}$? What about $\overline{PT}$ and $\overline{RT}$?



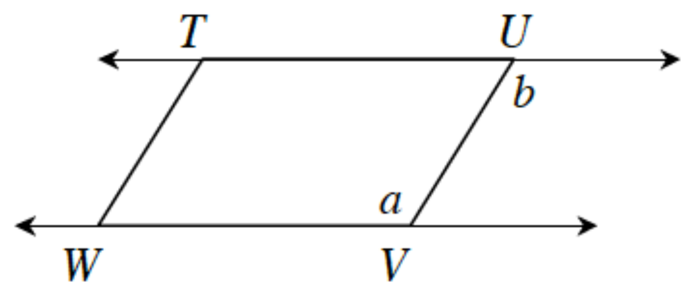
b. **CONJECTURE:** Use your observations from part (a) to write a conjecture on the relationship of the diagonals of a rhombus.

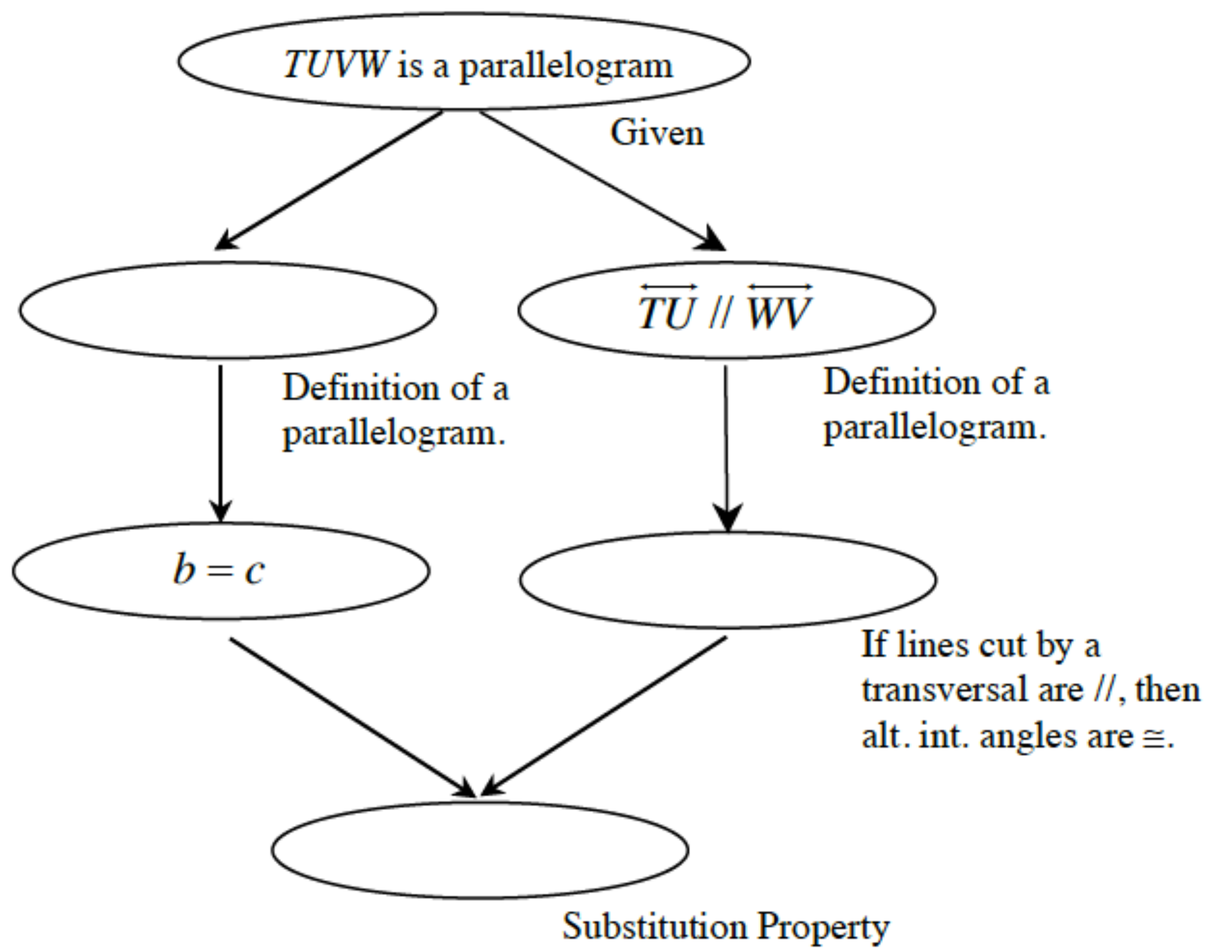
c. **PROVE:** Write a flowchart proof that proves your conjecture from part (b). Remember that to be convincing, you need to justify each statement with a reason. To help guide your discussion, consider the questions below. Which triangles should you use? Find two triangles that involve the segments $\overline{ST}$, $\overline{QT}$, $\overline{PT}$, and $\overline{RT}$.

- How can you prove these triangles are congruent? Create a flowchart proof with reasons to prove these triangles must be congruent.
- How can you use the congruent triangles to prove your conjecture from part (b)? Extend your flowchart proof to include this reasoning and prove your conjecture.

7-63. There are often many ways to prove a conjecture. You have rotated triangles to create parallelograms and used congruent parts of congruent triangles to justify that opposite sides are parallel. But is there another way?

Ansel wants to prove the conjecture "*If a quadrilateral is a parallelogram, then opposite angles are congruent.*" He started by drawing parallelogram *TUVW* at right. Copy and complete his flowchart. Make sure that each statement has a reason.





7-64. Think about the new facts you have proven about rhombi during this lesson. On your Theorem Toolkit (Lesson 7.2.1A Resource Page), record each new theorem you have proven about the angles and diagonals of a rhombus. Include clearly labeled diagrams to illustrate your findings.

Methods and Meanings

MATH NOTES

Exponential Functions

An **exponential function** has the general form $y = a \cdot b^x$, where a is the **initial value** (the y-intercept) and b is the **multiplier** (the growth). Be careful: The independent variable x has to be in the exponent. For example, $y = x^2$ is *not* an exponential equation, even though it has an exponent.

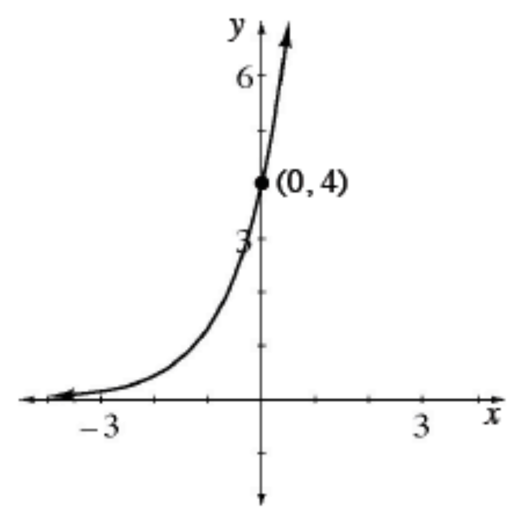
For example, in the multiple representations below, the y-intercept is $(0, 4)$ and the growth factor is 3 because the y-value is increasing by multiplying by 3.

$y = 4 \cdot 3^x$

x	y
–3	$\frac{4}{3^3}$ or $\frac{4}{27}$
–2	$\frac{4}{3^2}$ or $\frac{4}{9}$
–1	$\frac{4}{3}$
0	4
1	12
2	36
3	108

$\times 3$

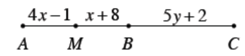
$\times 3$



To increase or decrease a quantity by a percentage, use the multiplier for that percentage. For example, the multiplier for an increase of 7% is 100% + 7% = 1.07. The multiplier for a decrease of 7% is 100% – 7% = 0.93.

Review & Preview

7-65. Point M is the midpoint of $\overline{AB}$ and B is the midpoint of $\overline{AC}$. What are the values of x and y? Show all work and reasoning.



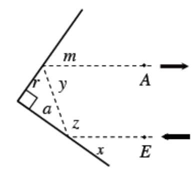
7-66. Read the Math Notes box in this lesson and then answer following questions. The cost of large flat-screen televisions is decreasing 20% per year.

a. What is the multiplier?

b. If a 50-inch flat-screen now costs \$1200, what will it cost in three years?

c. At the same rate, what did it cost two years ago?

7-67. A light directed from point E is pointed at a hinged mirror with right angle as shown at right.



a. If $\angle x$ measures 36°, find the measures of $\angle a$, $\angle r$, $\angle m$, $\angle y$, and $\angle z$.

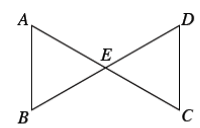
b. Why must the arrows at points A and E be parallel?

7-68. On graph paper, graph quadrilateral $MNPQ$ if $M(-3, 6)$, $N(2, 8)$, $P(1, 5)$, and $Q(-4, 3)$.

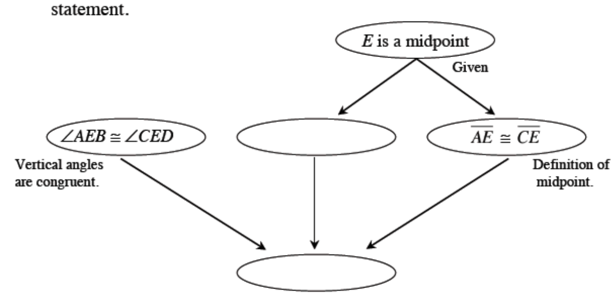
a. What shape is $MNPQ$? Show how you know.

b. Use the function $x \rightarrow x, y \rightarrow -y$ to reflect $MNPQ$ across the x-axis and create $M'N'P'Q'$. What are the coordinates of P'?

7-69. Jester started to prove that the triangles at right are congruent. He was only told that point E is the midpoint of segments $\overline{AC}$ and $\overline{BD}$.



Copy and complete his flowchart below. Be sure that a reason is provided for every statement.



7-70. For a school fair, Donny is going to design a spinner with red, white, and blue regions. Since he has a certain proportion of three types of prizes, he wants the P(red) = 40% and P(white) = 10%.

a. If the spinner only has red, white, and blue regions, then what is P(blue)? Explain how you know.

b. Find the central angles of this spinner if it has only three sections. Then draw a sketch of the spinner. Be sure to label the regions accurately.

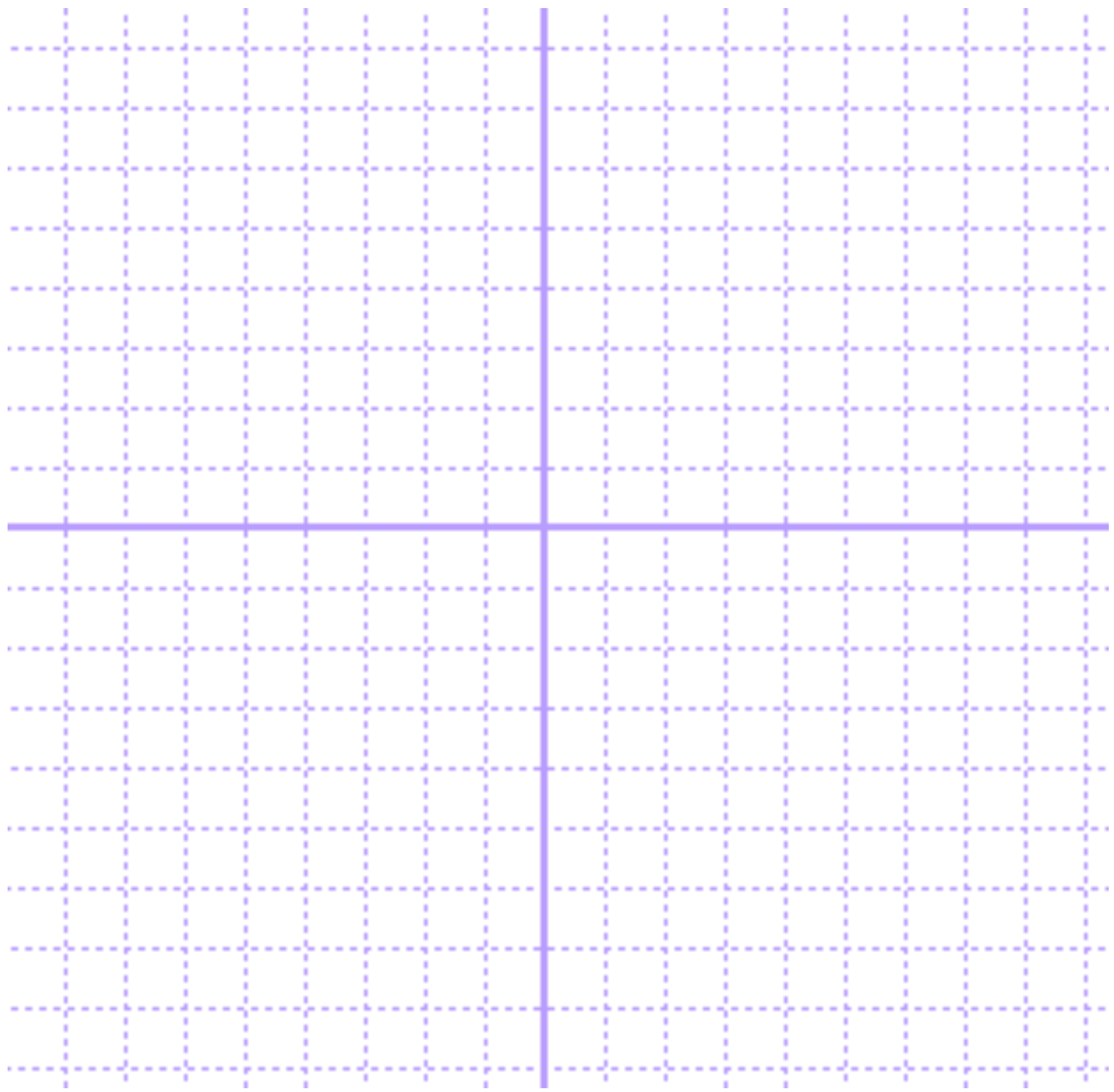
c. Is there a different spinner that has the same probabilities? If so, sketch another spinner that has the same probabilities. If not, explain why there is no other spinner with the same probabilities.

7-71. On graph paper, graph and shade the solutions for the inequality below.

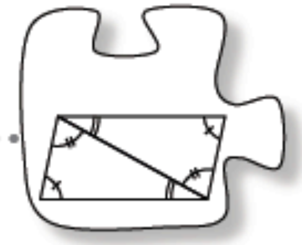
$$y < -\frac{2}{3}x + 5$$

7-71. On graph paper, graph and shade the solutions for the inequality below.

$$y < -\frac{2}{3}x + 5$$



7.2.3 What else can be proved?



More Proofs with Congruent Triangles

In Lessons 7.2.1 and 7.2.2, you used congruent triangles to learn more about parallelograms, kites, and rhombi. You now possess the tools to do the work of a geometer (someone who studies geometry): to discover and prove new properties about the sides and angles of shapes.

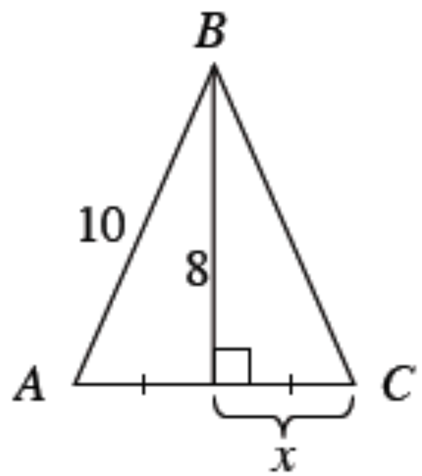
As you investigate these shapes, focus on proving your ideas. Remember to ask yourself and your teammates questions such as, "*Why does that work?*" and "*Is it always true?*" Decide whether your argument is convincing and work with your team to provide all of the necessary justification.

7-72. Carla decided to turn her attention to rectangles. Knowing that a rectangle is defined as a quadrilateral with four right angles, she drew the diagram at right.

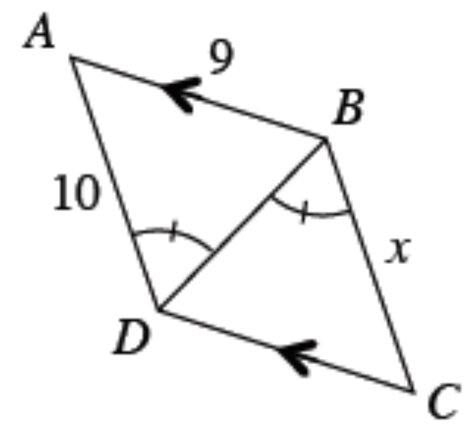
After some exploration, she conjectured that all rectangles are also parallelograms. Help her prove that her rectangle *ABCD* must be a parallelogram. That is, prove that the opposite sides must be parallel. Then add this theorem to your Theorem Toolkit (Lesson 7.2.1A Resource Page).

7-73. For each diagram below, find the value of x, if possible. If the triangles are congruent, state which triangle congruence condition was used. If the triangles are not congruent or if there is not enough information, state, "Cannot be determined."

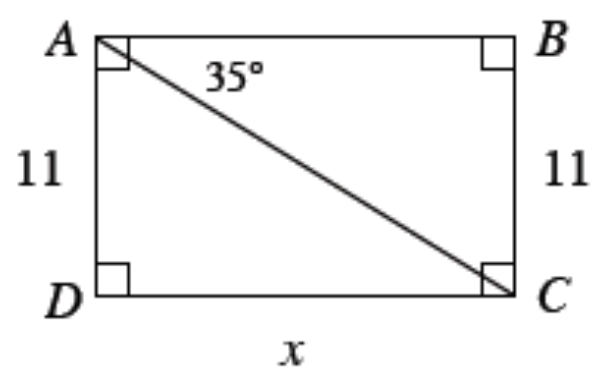
a. ABC below is a triangle.



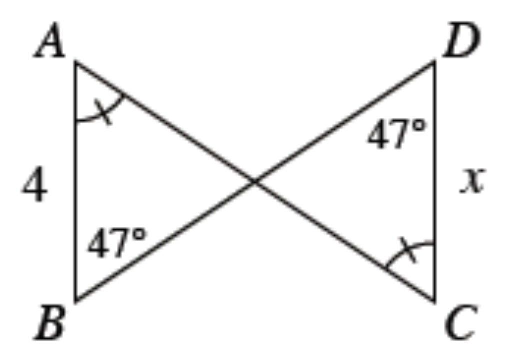
b.



c.



d. $\overline{AC}$ and $\overline{BD}$ are straight line segments.



7-74. With the class or your team, create a flowchart to prove your answer to part (b) of problem 7-73. That is, prove that $\overline{AD} \cong \overline{CB}$. Be sure to include a diagram for your proof and reasons for every statement. Make sure your argument is convincing and has no "holes."

MATH NOTES

METHODS AND MEANINGS

Definitions of Quadrilaterals

When proving properties of shapes, it is necessary to know exactly how a shape is defined. Below are the definitions of several quadrilaterals that you developed in Lesson 1.3.2 and that you will need to refer to in this chapter and the chapters that follow.

Quadrilateral: A closed four-sided polygon.

Kite: A quadrilateral with two distinct pairs of consecutive congruent sides.

Trapezoid: A quadrilateral with at least one pair of parallel sides.

Parallelogram: A quadrilateral with two pairs of parallel sides.

Rhombus: A quadrilateral with four sides of equal length.

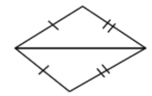
Rectangle: A quadrilateral with four right angles.

Square: A quadrilateral with four sides of equal length and four right angles.

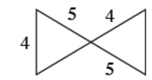
Review & Preview

7-75. Identify if each pair of triangles below is congruent or not. Remember that the diagram may not be drawn to scale. Justify your conclusion.

a.



b.



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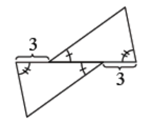

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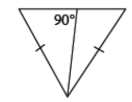
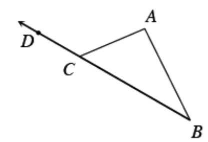


7-76. For either part (a), (c) or (d) of problem 7-75, create a flowchart to prove your conclusion. Remember to start with the given information and include a reason or justification for each "bubble" in your flowchart.

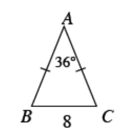
7-77. In the diagram at right, $\angle DCA$ is referred to as an **exterior angle** of ΔABC because it lies outside the triangle and is formed by extending a side of the triangle.



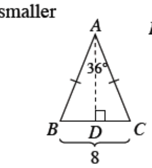
a. If $m\angle CAB = 46°$ and $m\angle ABC = 37°$, what is $m\angle DCA$? Show all work.

b. If $m\angle DCA = 135°$ and $m\angle ABC = 43°$, then what is $m\angle CAB$?

7-78. Tromika wants to find the area of the isosceles triangle at right.



a. She decided to start by drawing a height from vertex A to side $\overline{BC}$ as shown below. Will the two smaller triangles be congruent? In other words, is $\Delta ABC \cong \Delta ABD$? Why or why not?



b. What is $m\angle DAB$? BD?

c. Find AD. Show how you got your answer.

d. Find the area of ΔABC.

7-79. On graph paper, graph quadrilateral $ABCD$ if $A(0, 0)$, $B(6, 0)$, $C(8, 6)$, and $D(2, 6)$.

a. What is the best name for $ABCD$? Justify your answer.

b. Find the equation of the lines containing each diagonal. That is, find the equations of lines $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$.

7-80. It is often useful to estimate the value of a square root to determine if your answer is reasonable.

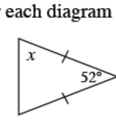
a. What would be a reasonable estimate of $\sqrt{68}$? Explain your thinking. After you have made an estimate, check your estimation with a calculator.

b. Repeat this process to estimate the values below.

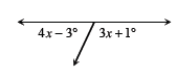
(1) $\sqrt{5}$ (2) $\sqrt{85}$ (3) $\sqrt{50}$ (4) $\sqrt{22}$

7-81. For each diagram below, solve for x. Show all work.

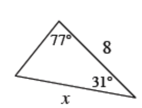
a.



b.



c.



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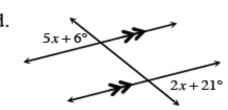
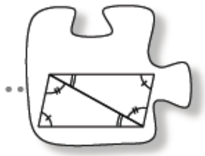


7.2.4 What else can I prove?



More Properties of Quadrilaterals

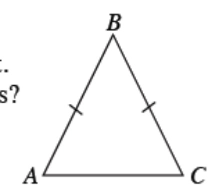
Today you will work with your team to apply what you have learned to other shapes. Remember to ask yourself and your teammates questions such as, "*Why does that work?*" and "*Is it always true?*" Decide whether your argument is convincing and work with your team to provide all of the necessary justification. By the end of this lesson, you should have a well-crafted mathematical argument proving something new about a familiar quadrilateral.

7-82. WHAT ELSE CAN CONGRUENT TRIANGLES TELL US?

Your Task: For each situation below, determine how congruent triangles can tell you more information about the shape. Then prove your conjecture using a flowchart. Be sure to provide a reason for each statement. For example, stating "$m\angle A = m\angle B$" is not enough. You must give a convincing reason, such as "*Because vertical angles are equal*" or "*Because it is given in the diagram.*" Use your triangle congruence conditions to help prove that the triangles are congruent.

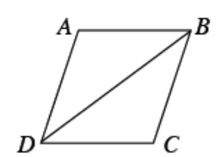
Later, your teacher will select one of these flowcharts for you to place on a poster. On your poster, include a diagram and all of your statements and reasons. Clearly state what you are proving and help the people who look at your poster understand your logic and reasoning.

a. In Chapter 1, you used the symmetry of an isosceles triangle to show that the base angles must be congruent. How can you prove this result using congruent triangles?

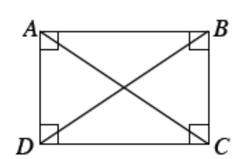


Assume that $\overline{AB} \cong \overline{CB}$ for the triangle at right. With your team, decide how to split ΔABC into two triangles that you can show are congruent to show that $\angle BAC \cong \angle BCA$.

b. What can congruent triangles tell us about the diagonals and angles of a rhombus? Examine the diagram of the rhombus at right. With your team, decide how to prove that the diagonals of a rhombus bisect the angles. That is, prove that $\angle ABD \cong \angle CBD$.



c. What can congruent triangles tell us about the diagonals of a rectangle? Examine the rectangle at right. Using the fact that the opposite sides of a rectangle are parallel (which you proved in problem 7-72), prove that the diagonals of the rectangle are congruent. That is, prove that $AC = BD$.

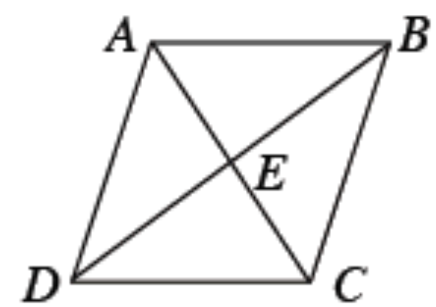


MATH NOTES

METHODS AND MEANINGS

Diagonals of a Rhombus

A **rhombus** is defined as a quadrilateral with four sides of equal length. In addition, you proved in problem 7-62 that the diagonals of a rhombus are perpendicular bisectors of each other.



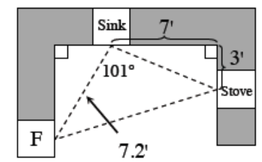
For example, in the rhombus at right, E is a midpoint of both $\overline{AC}$ and $\overline{DB}$. Therefore, $AE = CE$ and $DE = BE$. Also, $m\angle AEB = m\angle BEC = m\angle CED = m\angle DEA = 90^\circ$.

In addition, you proved in problem 7-82 that the diagonals bisect the angles of the rhombus. For example, in the diagram above, $m\angle DAE = m\angle BAE$.

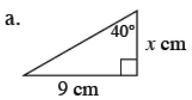
Review & Preview

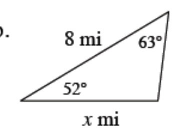
7-83. Use Tromika's method from problem 7-78 to find the area of an equilateral triangle with side length 12 units. Show all work.

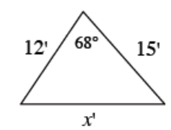
7-84. The guidelines set forth by the National Kitchen & Bath Association recommends that the perimeter of the triangle connecting the refrigerator (F), stove, and sink of a kitchen be 26 feet or less. Lashayia is planning to renovate her kitchen and has chosen the design at right. Does her design conform to the National Kitchen and Bath Association's guidelines? Show how you got your answer.



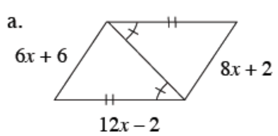
7-85. Examine the triangles below. For each, solve for x and name which tool you use. Show all work.

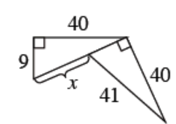
a. 

b. 

c. 

7-86. For each figure below, determine if the two smaller triangles in each figure are congruent. If so, create a flowchart to explain why. Then, solve for x. If the triangles are not congruent, explain why not.

a. 

b. 

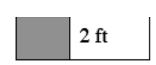
7-87. The diagonals of a rhombus are 6 units and 8 units long. What is the area of the rhombus? Draw a diagram and show all reasoning.

7-88. Kendrick is frantic. He remembers that several years ago he buried his Amazing Electron Ring in his little sister's sandbox, but he cannot remember where. A few minutes ago he heard that someone is willing to pay $1000 for it. He has his shovel ar

7-89. What is the 50th term in this sequence?

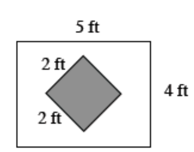
17, 14, 11, 8, …

a. The sar 5 feet, time to search in the 2 foot-square shaded region, what is the probability that he will find the ring?



b. What is the probability that he will not find the ring? Explain how you found your answer.

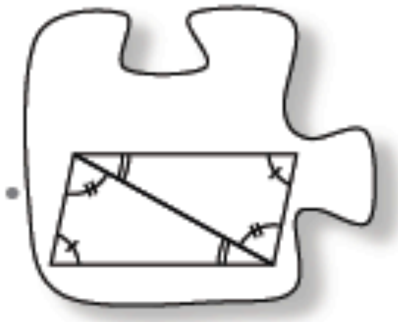
c. Kendrick decides instead to dig in the square region shaded at right. Does this improve his chances for finding the ring? Why or why not?



7-89. What is the 50th term in this sequence?

17, 14, 11, 8, …

7.2.5 How else can I write it?

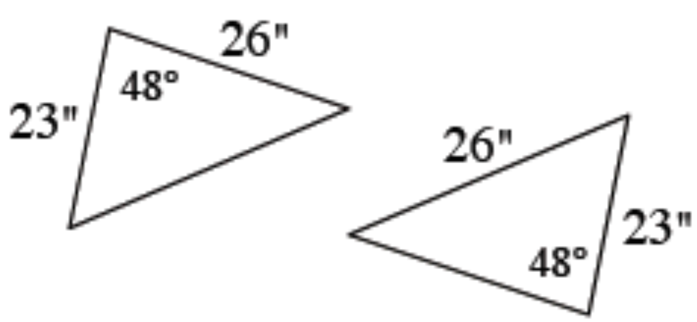


Two-Column Proofs

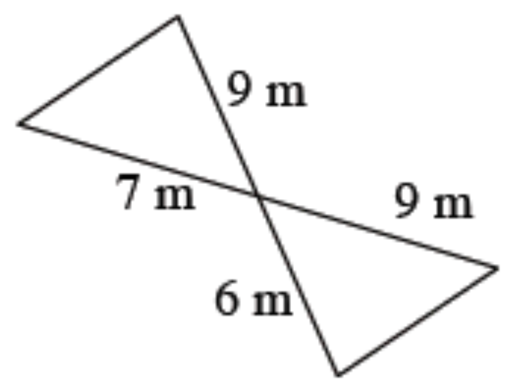
Today you will continue to work with constructing a convincing argument, otherwise known as writing a proof. In this lesson, you will use what you know about flowchart proofs to write a convincing argument using another format, called a two-column proof.

7-90. The following pairs of triangles are not necessarily congruent even though they appear to be. Use the information provided in the diagram to show why. Justify your statements.

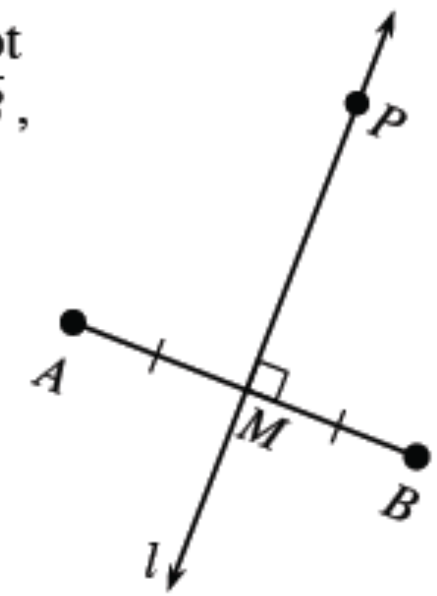
a.



b.

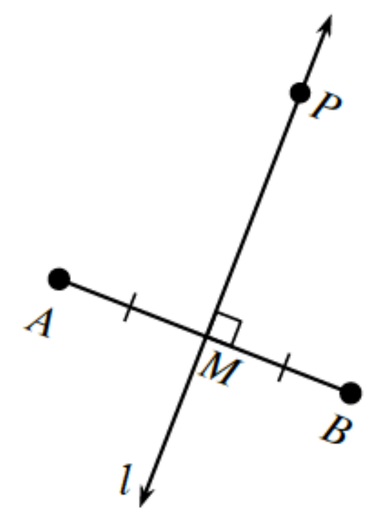


7-91. Write a flowchart to prove that if point P on line l (not on $\overline{AB}$) is a point on the perpendicular bisector of $\overline{AB}$, then $\overline{PA} \cong \overline{PB}$. That is, point P is the same distance from points A and B *(called "equidistant" in mathematics)*. Assume the intersection of $\overline{AB}$ and line l is point M as shown in the diagram.



7-92. Another way to organize a proof is called a **two-column proof**. Instead of using arrows to indicate the order of logical reasoning, this style of proof lists statements and reasons in a linear order, first to last, in columns.

The proof from problem 7-91 has been converted to a two-column proof below. Copy and complete the proof on your paper using your statements and reasons from problem 7-91.



If: M is on $\overline{AB}$ and $\overleftrightarrow{PM}$ is the perpendicular bisector of $\overline{AB}$

Prove: $\overline{PA} \cong \overline{PB}$

Statements	**Reasons** **(This statement is true because…)**
Point M is on $\overline{AB}$ and $\overleftrightarrow{PM}$ is the perpendicular bisector of $\overline{AB}$.	Given
$m\angle PMA = m\angle PMB = 90°$	Definition of perpendicular and angles with the same measure are congruent.
	Definition of a bisector.
$\overline{PM} \cong \overline{PM}$	

7-93. Examine the posters of flowchart proofs from problem 7-82. Convert each flowchart proof to a two-column proof. Remember that one column must contain the ordered statements of fact while the other must provide the reason (or justification) explaining why that fact must be true.

7-94. So far in Section 7.2, you have proven many special properties of quadrilaterals and other shapes. Remember that when a conjecture is proven, it is called a theorem. For example, once you proved the relationship between the lengths of the sides of a right triangle, you were able to refer to that relationship as the Pythagorean Theorem. Find your Theorem Toolkit (Lesson 7.2.1A Resource Page) and make sure it contains all of the theorems you and your classmates have proven so far about various quadrilaterals. Be sure that your records include diagrams for each statement.

7-95. LEARNING LOG

Reflect on the new proof format you learned today. Compare it to the flowchart proof format that you have used earlier. What are the strengths and weaknesses of each style of proof? Which format is easier for you to use? Which is easier to read? Title this entry "Two-Column Proofs" and label it with today's date.

Review & Preview

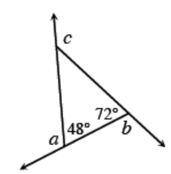
7-96. Suppose you know that $\Delta TAP \cong \Delta DOG$ where $TA = 14$, $AP = 18$, $TP = 21$, and $DG = 2y + 7$.

a. On your paper, draw a reasonable sketch of ΔTAP and ΔDOG.

b. Find y. Show all work.

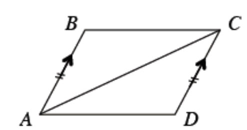
7-97. Graph the equation $y = -\frac{3}{2}x + 6$ on graph paper. Label the points where the line intersects the x- and y-axes.

7-98. $\angle a$, $\angle b$, and $\angle c$ are exterior angles of the triangle at right. Find $m\angle a$, $m\angle b$, and $m\angle c$. Then find $m\angle a + m\angle b + m\angle c$.



7-99. The principal's new car cost \$35,000 but in three years it will only be worth \$21,494. What is the annual percent of decrease?

7-100. What else can you prove about parallelograms? Prove that if a pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral must be a parallelogram. For example, for the quadrilateral $ABCD$ at right, given that $\overline{AB} \parallel \overline{CD}$ and $\overline{AB} \cong \overline{CD}$, show that $\overline{BC} \parallel \overline{AD}$. Organize your reasoning in a flowchart. Then record your theorem in your Theorem Toolkit (Lesson 7.2.1A Resource Page).



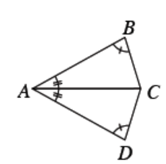
7-101. As Ms. Dorman looked from the window of her third-story classroom, she noticed Pam in the courtyard. Ms. Dorman's eyes were 52 feet above ground and Pam was 38 feet from the building. Draw a diagram of this situation. What is the angle at which Ms. Dorman had to look down, that is, what is the angle of depression? (Assume that Ms. Dorman was looking at the spot on the ground below Pam.)

7-102. For each pair of triangles below, determine if the triangles are congruent. If the triangles are congruent,

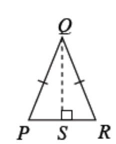
- complete the correspondence statement,
- state the congruence property,
- and record any other ideas you use that make your conclusion true.

Otherwise, explain why you cannot conclude that the triangles are congruent. Note that the figures are not necessarily drawn to scale.

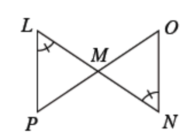
a. $\Delta ABC \cong \Delta$_____



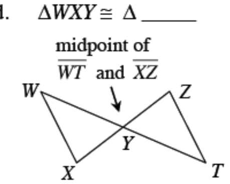
b. $\Delta SQP \cong \Delta$_____



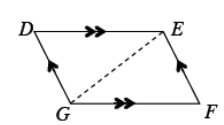
c. $\Delta PLM \cong \Delta$_____



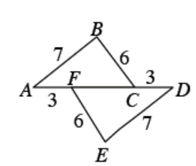
d. $\Delta WXY \cong \Delta$_____



e. $\Delta EDG \cong \Delta$_____



f. $\Delta ABC \cong \Delta$_____



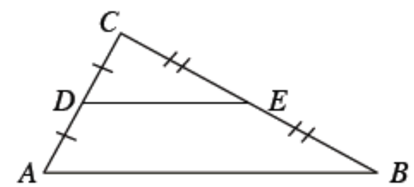
7.2.6 What can I prove?

Explore-Conjecture-Prove

So far, congruent triangles have helped you to discover and prove many new facts about triangles and quadrilaterals. But what else can you discover and prove? Today your work will mirror the real work of professional mathematicians. You will investigate relationships, write a conjecture based on your observations, and then prove your conjecture.

7-103. TRIANGLE MIDSEGMENT THEOREM

As Sergio was drawing shapes on his paper, he drew a line segment that connected the midpoints of two sides of a triangle. This is called the **midsegment** of a triangle. *"I wonder what we can find out about this midsegment,"* he said to his team. Examine his drawing at right.



a. **EXPLORE:** Examine the diagram of ΔABC, drawn to scale above. How do you think $\overline{DE}$ is related to $\overline{AB}$? How do their lengths seem to be related?

b. **CONJECTURE:** Write a conjecture about the relationship between segments $\overline{DE}$ and $\overline{AB}$.

c. **PROVE:** Sergio wants to prove that $AB = 2BE$. However, he does not see any congruent triangles in the diagram. How are the triangles in this diagram related? How do you know? Prove your conclusion with a flowchart.

d. What is the common ratio between side lengths in the similar triangles? Use this to write a statement relating lengths DE and AB.

e. Now Sergio wants to prove that $\overline{DE} \parallel \overline{AB}$. Use the similar triangles to find all the pairs of equal angles you can in the diagram. Then use your knowledge of angle relationships to make a statement about parallel segments.

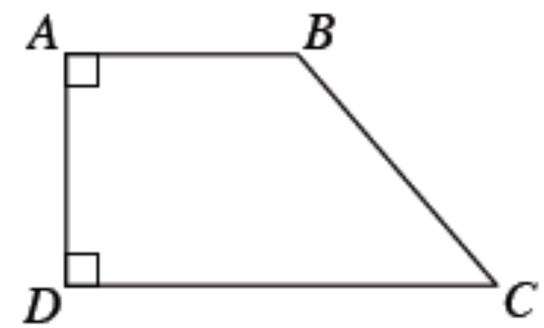
7-104. The work you did in problem 7-103 mirrors the work of many professional mathematicians. In the problem, Sergio examined a geometric shape and thought there might be something new to learn. You then helped him by finding possible relationships and writing a conjecture. Then, to find out if the conjecture was true for all triangles, you wrote a convincing argument (or proof). This process is summarized in the diagram below.

Explore		Conjecture		Prove
Use any tools that are available to explore a shape and discover any possible relationships.	⟹	Write a conditional statement or arrow diagram based on your observations from the investigation.	⟹	Convince yourself (and others) that your conjecture is always true with a convincing argument.

Discuss this process with the class and describe when you have used this process before (either in this class or outside of class). Why do mathematicians rely on this process?

7-105. RIGHT TRAPEZOIDS

Consecutive angles of a polygon occur at opposite ends of a side of the polygon. What can you learn about a quadrilateral with two consecutive right angles?



a. **EXPLORE:** Examine the quadrilateral at right with two consecutive right angles. What do you think is true about $\overline{AB}$ and $\overline{DC}$?

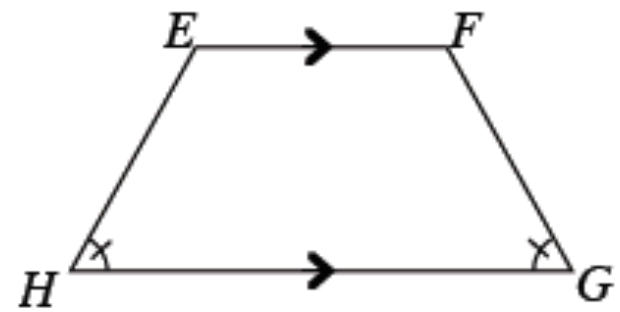
b. **CONJECTURE:** Write a conjecture about what type of quadrilateral has two consecutive right angles. Write your conjecture in conditional ("If…, then…") form.

c. **PROVE:** Prove that your conjecture from part (b) is true for all quadrilaterals with two consecutive right angles. Write your proof using the two-column format introduced in Lesson 7.2.4. Hint: Look for angle relationships.

d. The quadrilateral you worked with in this problem is called a **right trapezoid**. Are all quadrilaterals with two right angles a right trapezoid?

7-106. ISOSCELES TRAPEZOIDS

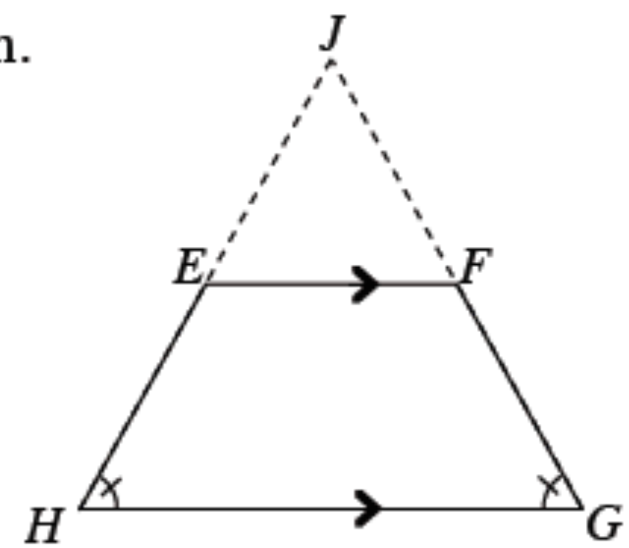
An **isosceles trapezoid** is a trapezoid with a pair of congruent base angles. What can you learn about the sides of an isosceles trapezoid?



a. **EXPLORE:** Examine trapezoid *EFGH* at right. How do the non-parallel side lengths appear to be related?

b. **CONJECTURE:** Write a conjecture about side lengths in an isosceles trapezoid. Write your conjecture in conditional ("If…, then…") form.

c. **PROVE:** Now prove that your conjecture from part (b) is true for all isosceles trapezoids. Write your proof using the two-column format introduced in Lesson 7.2.5. To help you get started, the isosceles trapezoid is shown at right with its sides extended to form a triangle.



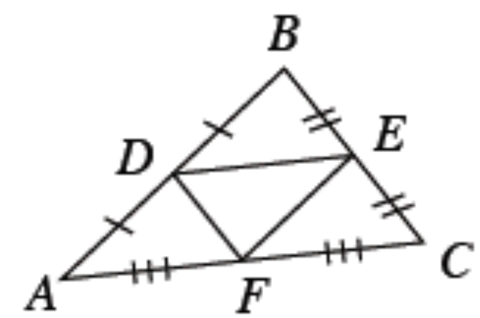
7-107. Add the theorems you have proved in this lesson to your Theorem Toolkit (Lesson 7.2.1A Resource Page). Be sure to include diagrams for each statement.

METHODS AND MEANINGS

MATH NOTES

Triangle Midsegment Theorem

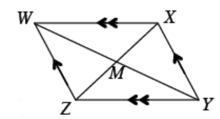
A **midsegment** of a triangle is a segment that connects the midpoints of any two sides of a triangle. Every triangle has three midsegments, as shown at right.



A midsegment between two sides of a triangle is half the length of and parallel to the third side of the triangle. For example, in ΔABC at right, $\overline{DE}$ is a midsegment, $\overline{DE} \parallel \overline{AC}$, and $DE = \frac{1}{2}AC$.

Review & Preview

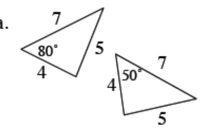
7-108. What else can you prove about parallelograms? Prove that the diagonals of a parallelogram bisect each other. For example, assuming that quadrilateral $WXYZ$ at right is a parallelogram, prove that $\overline{WM} \cong \overline{YM}$ and $\overline{ZM} \cong \overline{XM}$. Organize your reasoning in a flowchart. Then record your theorem in your Theorem Toolkit (Lesson 7.2.1A Resource Page).



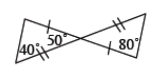
7-109. One way a shape can be special is to have two congruent sides. For example, an isosceles triangle is special because it has a pair of sides that are the same length. Think about all the shapes you know and list the other special properties shapes can have. List as many as you can. Be ready to share your list with the class at the beginning of Lesson 7.3.1.

7-110. Carefully examine each diagram below and explain why the geometric figure cannot exist. Support your statements with reasons. If a line looks straight, assume that it is.

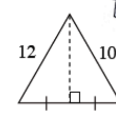
a.



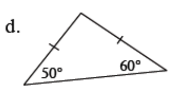
b.



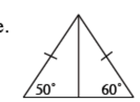
c.



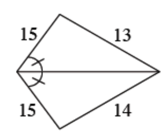
d.



e.



f.



7-111. Remember that if a triangle has two equal sides, it is called isosceles. Decide whether each triangle formed by the points below is isosceles. Explain how you decided.

a. (6, 0), (0, 6), (6, 6)

b. (–3, 7), (–5, 2), (–1, 2)

c. (4, 1), (2, 3), (9, 2)

d. (1, 1), (5, –3), (1, –7)

7-112. For each pair of numbers, find the number that is exactly halfway between them.

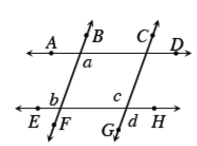
a. 9 and 15

b. 3 and 27

c. 10 and 21

7-113. Penn started the proof below to show that if $\overline{AD} \parallel \overline{EH}$ and $\overline{BF} \parallel \overline{CG}$, then $a = d$. Unfortunately, he did not provide reasons for his proof. Copy his proof and provide a justification for each statement.

Statements	Reasons
1. $\overline{AD} \parallel \overline{EH}$ and $\overline{BF} \parallel \overline{CG}$	
2. $a = b$	
3. $b = c$	
4. $a = c$	
5. $c = d$	
6. $a = d$	



7-114. After finding out that her kitchen does not conform to industry standards, Lashayia is back to the drawing board (see problem 7-84). Where can she locate her sink along her top counter so that its distance from the stove and refrigerator is as small as possible? And will this location keep her perimeter below 26 feet? Show all work.