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Special Quadrilaterals and Proof

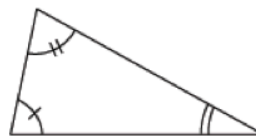
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7-71. On graph paper, graph and shade the solutions for the inequality below.

$$y < -\frac{2}{3}x + 5$$

7-49. Carla is thinking about parallelograms and wondering if there are as many special properties for parallelograms as there are for triangles. She remembers that it is possible to create a shape that looks like a parallelogram by rotating a triangle about the midpoint of one of its sides.

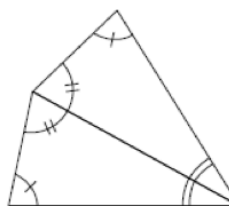
- a. Carefully trace the triangle at right onto tracing paper. Be sure to copy the angle markings as well. Then rotate the triangle about a midpoint of a side to make a shape that looks like a parallelogram.



- b. Is Carla's shape truly a parallelogram? Use the angles to convince your teammates that the opposite sides must be parallel. Then write a convincing argument.
- c. What else can the congruent triangles tell you about a parallelogram? Look for any relationships you can find between the angles and sides of a parallelogram.

7-51. Kip is confused. He put his two triangles from problem 7-49 together as shown at right, but he did not get a parallelogram.

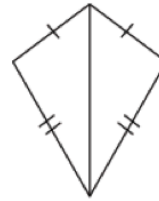
- a. What shape did he make? Justify your conclusion.
- b. What transformation(s) did Kip use to form his shape?



- c. What do the congruent triangles tell you about the angles of this shape?

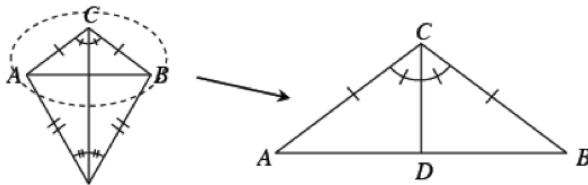
7-52. KITES

Kip shared his findings about his kite with his teammate, Carla, who wants to learn more about the diagonals of a kite. Carla quickly sketched the kite at right onto her paper with a diagonal showing the two congruent triangles.



- a. **EXPLORE:** Trace this diagram onto tracing paper and carefully add the other diagonal. Then, with your team, consider how the diagonals may be related. Use tracing paper to help you explore the relationships between the diagonals. If you make an observation you think is true, move on to part (b) and write a conjecture.
- b. **CONJECTURE:** If you have not already done so, write a conjecture based on your observations in part (a).

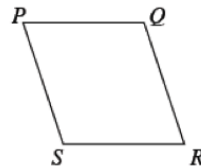
- b. **CONJECTURE:** If you have not already done so, write a conjecture based on your observations in part (a).
- c. **PROVE:** When she drew the second diagonal, Carla noticed that four new triangles appeared. “If any of these triangles are congruent, then they may be able to help us prove our conjecture from part (b),” she said. Examine  $\triangle ABC$  below. Are  $\triangle ACD$  and  $\triangle BCD$  congruent? Create a flowchart proof like the one from problem 7-50 to justify your conclusion.



- d. Now extend your proof from part (c) to prove your conjecture from part (b).

7-61. Audrey has a favorite quadrilateral – the rhombus. Even though a rhombus is defined as having four congruent sides, she suspects that the sides of a rhombus have other special properties.

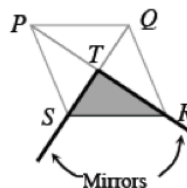
- a. **EXPLORE:** Draw a rhombus like the one at right on your paper. Mark the side lengths equal.
- b. **CONJECTURE:** What else might be special about the sides of a rhombus? Write a conjecture.



- c. **PROVE:** Audrey knows congruent triangles can help prove other properties about quadrilaterals. She starts by adding a diagonal  $\overline{PR}$  to her diagram so that two triangles are formed. Add this diagonal to your diagram and prove that the created triangles are congruent. Then use a flowchart with reasons to show your logic. Be prepared to share your flowchart with the class.
- d. How can the triangles from part (c) help you prove your conjecture from part (b) above? Discuss with the class how to extend your flowchart to convince others. Be sure to justify any new statements with reasons.

7-62. Now that you know the opposite sides of a rhombus are parallel, what else can you prove about a rhombus? Consider this as you answer the questions below.

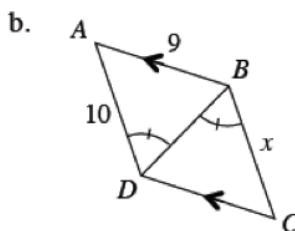
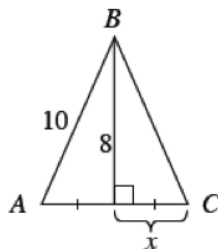
- a. **EXPLORE:** Remember that in Lesson 7.1.4, you explored the shapes that could be formed with a hinged mirror. During this activity, you used symmetry to form a rhombus. Think about what you know about the reflected triangles in the diagram. What do you think is true about the diagonals  $\overline{SQ}$  and  $\overline{PR}$ ? What is special about  $\overline{ST}$  and  $\overline{QT}$ ? What about  $\overline{PT}$  and  $\overline{RT}$ ?



- b. **CONJECTURE:** Use your observations from part (a) to write a conjecture on the relationship of the diagonals of a rhombus.
- c. **PROVE:** Write a flowchart proof that proves your conjecture from part (b). Remember that to be convincing, you need to justify each statement with a reason. To help guide your discussion, consider the questions below. Which triangles should you use? Find two triangles that involve the segments  $\overline{ST}$ ,  $\overline{QT}$ ,  $\overline{PT}$ , and  $\overline{RT}$ .

7-73. For each diagram below, find the value of  $x$ , if possible. If the triangles are congruent, state which triangle congruence condition was used. If the triangles are not congruent or if there is not enough information, state, "Cannot be determined."

- a.  $ABC$  below is a triangle.



7-82. WHAT ELSE CAN CONGRUENT TRIANGLES TELL US?

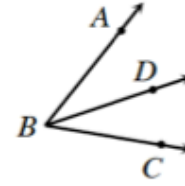
**Your Task:** For each situation below, determine how congruent triangles can tell you more information about the shape. Then prove your conjecture using a flowchart. Be sure to provide a reason for each statement. For example, stating " $m\angle A = m\angle B$ " is not enough. You must give a convincing reason, such as "*Because vertical angles are equal*" or "*Because it is given in the diagram.*" Use your triangle congruence conditions to help prove that the triangles are congruent.



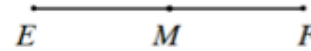
Later, your teacher will select one of these flowcharts for you to place on a poster. On your poster, include a diagram and all of your statements and reasons. Clearly state what you are proving and help the people who look at your poster understand your logic and reasoning.

7-54. Use the information given for each diagram below to solve for  $x$ . Show all work.

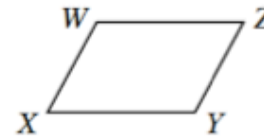
- a.  $\overline{BD}$  bisects  $\angle ABC$ . (Remember that this means it divides the angle into two equal parts.) If  $m\angle ABD = 5x - 10^\circ$  and  $m\angle ABC = 65^\circ$ , solve for  $x$ .



- b. Point  $M$  is a midpoint of  $\overline{EF}$ . If  $EM = 4x - 2$  and  $MF = 3x + 9$ , solve for  $x$ .

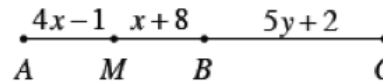


- c.  $WXYZ$  at right is a parallelogram. If  $m\angle W = 9x - 3^\circ$  and  $m\angle Z = 3x + 15^\circ$ , solve for  $x$ .



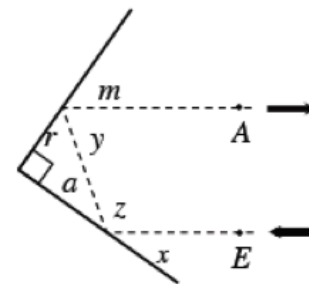
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- 7-65. Point  $M$  is the midpoint of  $\overline{AB}$  and  $B$  is the midpoint of  $\overline{AC}$ . What are the values of  $x$  and  $y$ ? Show all work and reasoning.



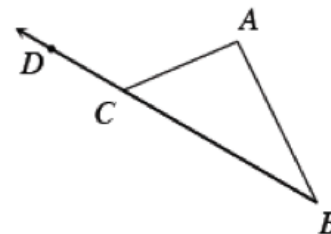
- 7-67. A light directed from point  $E$  is pointed at a hinged mirror with right angle as shown at right.

- a. If  $\angle x$  measures  $36^\circ$ , find the measures of  $\angle a$ ,  $\angle r$ ,  $\angle m$ ,  $\angle y$ , and  $\angle z$ .
- b. Why must the arrows at points  $A$  and  $E$  be parallel?

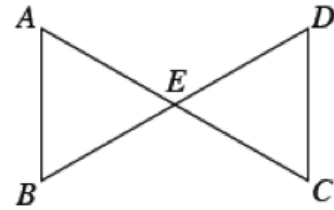


- 7-77. In the diagram at right,  $\angle DCA$  is referred to as an **exterior angle** of  $\triangle ABC$  because it lies outside the triangle and is formed by extending a side of the triangle.

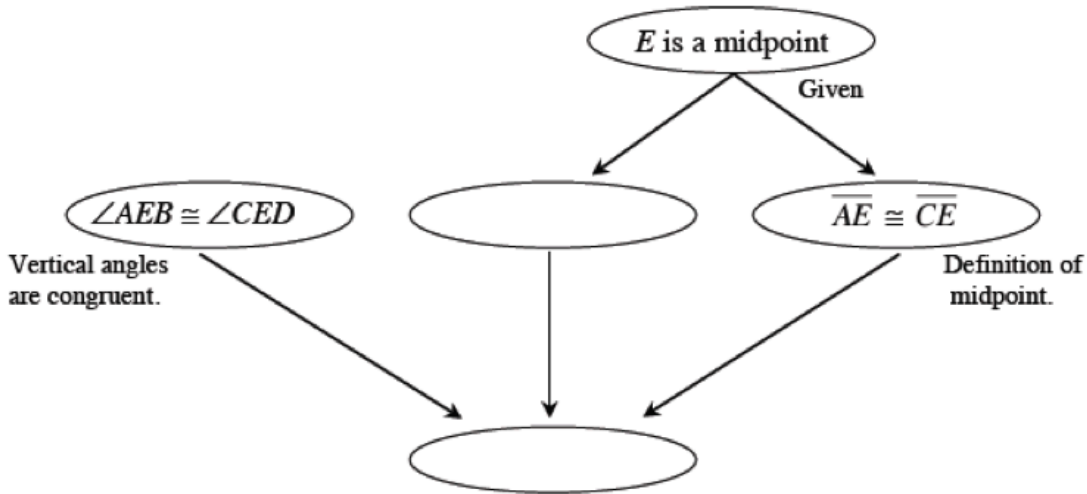
- a. If  $m\angle CAB = 46^\circ$  and  $m\angle ABC = 37^\circ$ , what is  $m\angle DCA$ ? Show all work.
- b. If  $m\angle DCA = 135^\circ$  and  $m\angle ABC = 43^\circ$ , then what is  $m\angle CAB$ ?



- 7-69. Jester started to prove that the triangles at right are congruent. He was only told that point  $E$  is the midpoint of segments  $\overline{AC}$  and  $\overline{BD}$ .

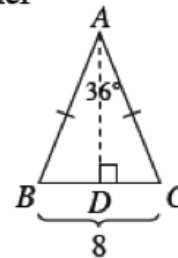
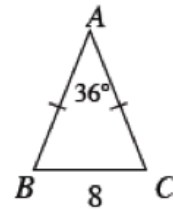


Copy and complete his flowchart below. Be sure that a reason is provided for every statement.

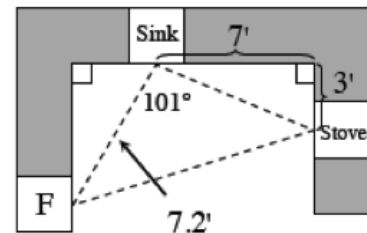


- 7-78. Tromika wants to find the area of the isosceles triangle at right.

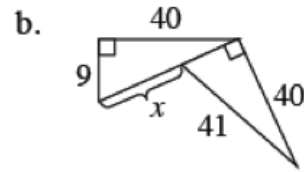
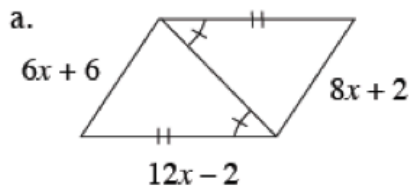
- She decided to start by drawing a height from vertex  $A$  to side  $\overline{BC}$  as shown below. Will the two smaller triangles be congruent? In other words, is  $\triangle ABC \cong \triangle ABD$ ? Why or why not?
- What is  $m\angle DAB$ ?  $BD$ ?
- Find  $AD$ . Show how you got your answer.
- Find the area of  $\triangle ABC$ .



- 7-84. The guidelines set forth by the National Kitchen & Bath Association recommends that the perimeter of the triangle connecting the refrigerator (F), stove, and sink of a kitchen be 26 feet or less. Lashayia is planning to renovate her kitchen and has chosen the design at right. Does her design conform to the National Kitchen and Bath Association's guidelines? Show how you got your answer.

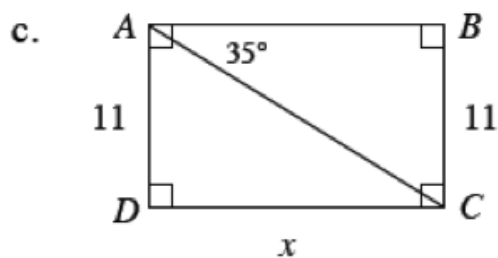


- 7-86. For each figure below, determine if the two smaller triangles in each figure are congruent. If so, create a flowchart to explain why. Then, solve for  $x$ . If the triangles are not congruent, explain why not.



- 7-87. The diagonals of a rhombus are 6 units and 8 units long. What is the area of the rhombus? Draw a diagram and show all reasoning.

For each diagram below, find the value of  $x$ , if possible. If the triangles are congruent, state which triangle congruence condition was used. If the triangles are not congruent or if there is not enough information, state, "Cannot be determined."



- d.  $\overline{AC}$  and  $\overline{BD}$  are straight line segments.

