Agenda: #68F Ch7-1 Properties of a Circle

1) Copy Title, Objectives, Warm Up (15min.)

- How to find the midpoint of a line segment.

a) List at least 4 parts of a circle.

b) Write down formulas for an area and a circumference of a circle.

c) Find a circumference and an area of a circle with a radius 10cm.

2) Q7-1 ~ 7-2 (Reading and filling out WS)

(15min)

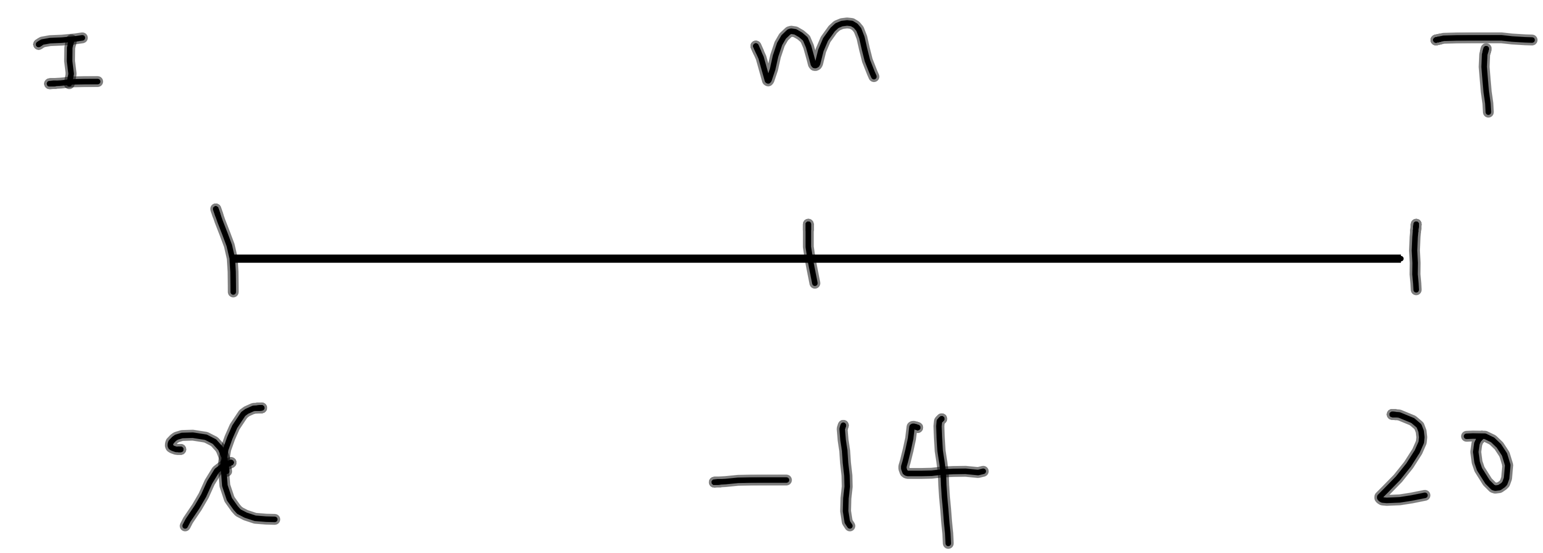
3) Q7-12 ~ 7-13 (Elbow Partner 15min)

4) Q7-14(Group and Sketching 15min)

5) (15min)

6) Closing Activities (15 min)

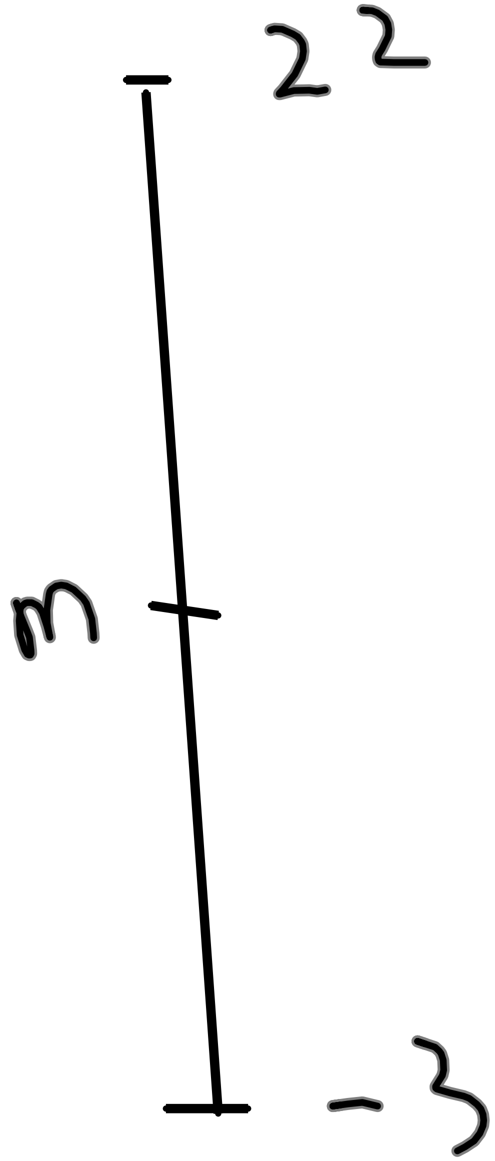
I … initial   Terminal



$$\frac{x+20}{2} = \frac{-14}{1}$$

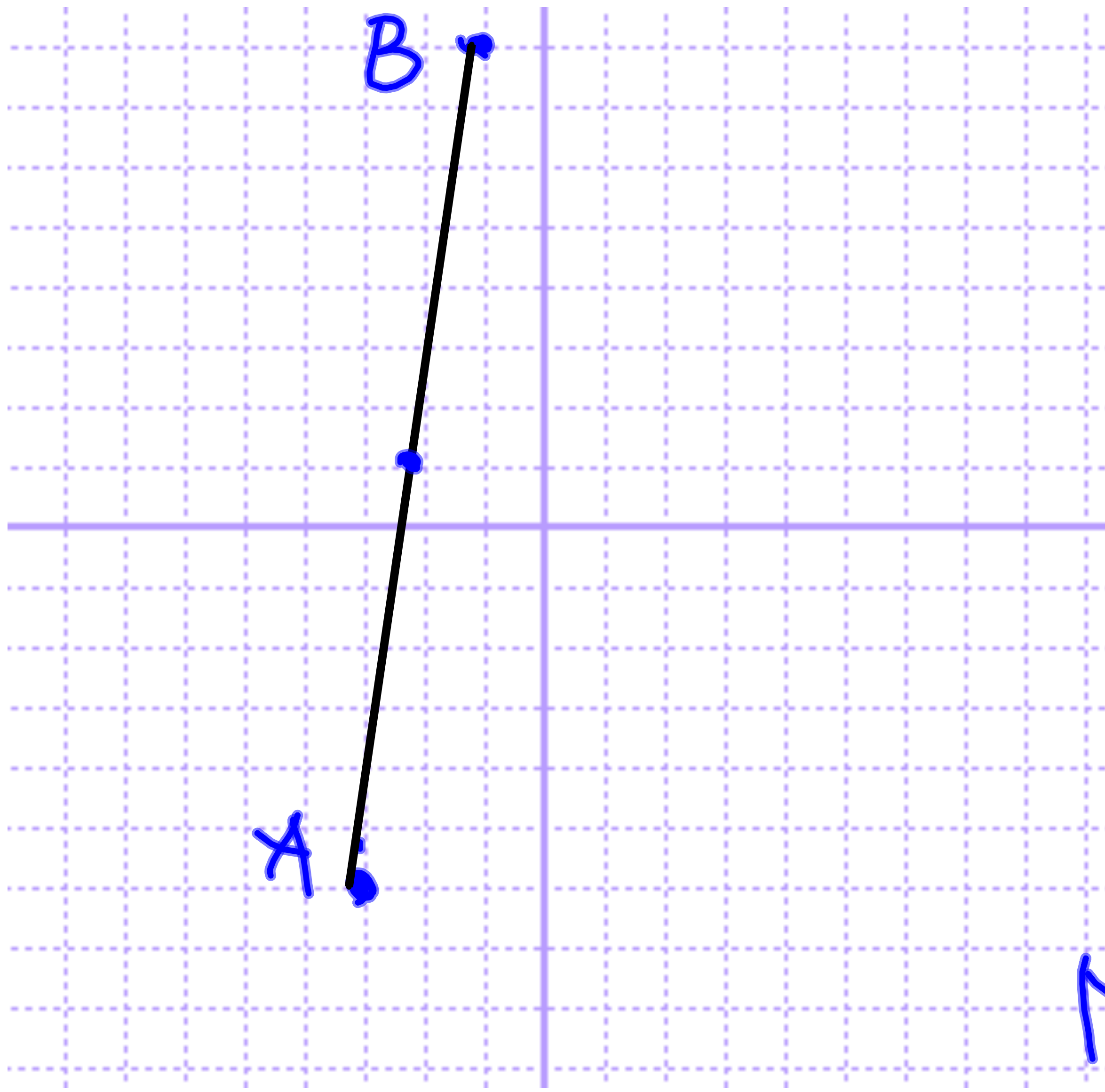
$$x+20 = -28$$

$$x = -48$$



$$\frac{22+(-3)}{2} = \frac{19}{2}$$

$$= 9.5$$



$A(-3, -6)$

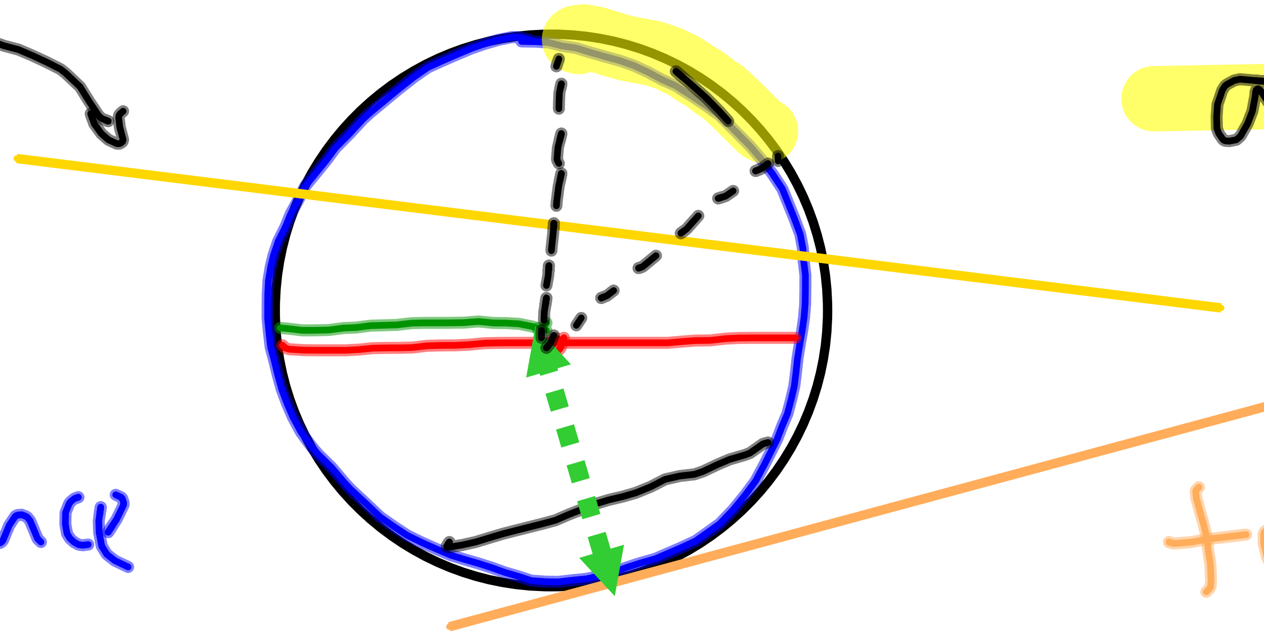
$B(-1, 8)$

$\frac{-4}{2}, \frac{2}{2}$

$M(-2, 1)$

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Secant line



arc

radius

circumference

tangent line

diameter

chord

b) Write down formulas for an area and a circumference of a circle.

$A = \pi r^2 \quad C = 2r\pi$

c) Find a circumference and an area of a circle with a radius 10cm.

$100\pi \, cm^2 \longleftarrow A = \pi(10)^2 = 100\pi$

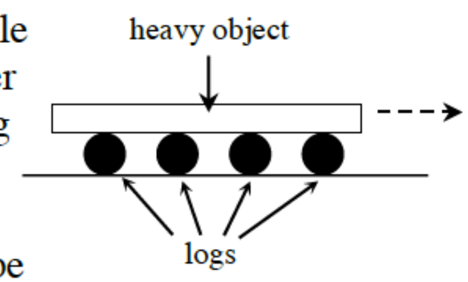
$20\pi \, cm \longleftarrow C = 2(10)\pi = 20\pi$

- The relationships of the sides, angles, and diagonals of special quadrilaterals, such as parallelograms, rectangles, kites, and rhombi (plural of rhombus).
- How to write a convincing proof in a variety of formats, such as a flowchart or two-column proof.
- How to find the midpoint of a line segment.
- How to use algebraic tools to explore quadrilaterals on coordinate axes.

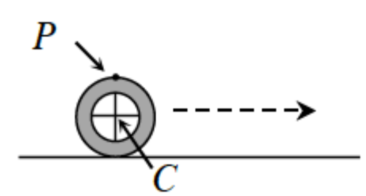
7-1. THE INVENTION OF THE WHEEL

One of the most important human inventions was the wheel. Many archeologists estimate that the wheel was probably first invented about 10,000 years ago in Asia. It was an important tool that enabled humans to transport very heavy objects long distances. Most people agree that impressive structures, such as the Egyptian pyramids, could not have been built without the help of wheels.

a. One of the earliest types of "wheels" used were actually logs. Ancient civilizations laid multiple logs on the ground, parallel to each other, under a heavy item that needed to be moved. As long as the logs had the same diameter and the road was even, the heavy object had a smooth ride. What is special about a circle that allows it to be used in this way? In other words, why do circles enable this heavy object to roll smoothly?

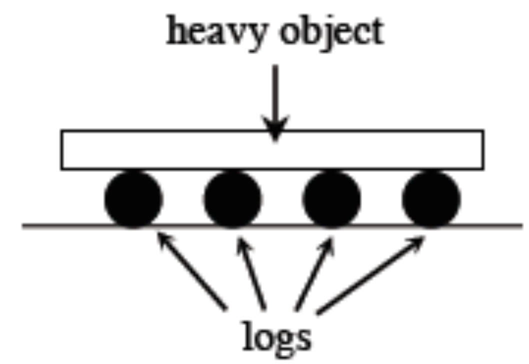


b. What happens to a point on a wheel as it turns? For example, as the wheel at right rolls along the line, what is the path of point P? Imagine a piece of gum stuck to a tire as it rolls. On your paper, draw the motion of point P. If you need help, find a coin or other round object and test this situation.



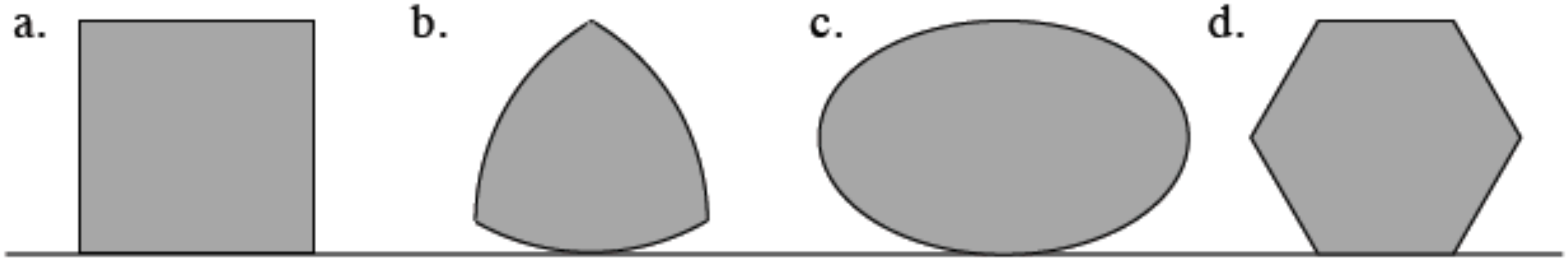
c. Now turn your attention to the center of the wheel (labeled C in the diagram above). As the wheel rolls along the line, what is the path of point C? Describe its motion. Why does that happen?

7-2. DO CIRCLES MAKE THE BEST WHEELS?

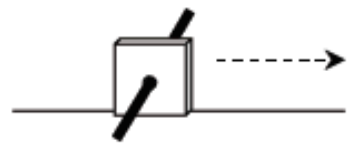


As you read in problem 7-1, ancient civilizations used circular logs to roll heavy objects. However, is a circle the only shape they could have chosen? Are there any other shapes that could rotate between a flat road and a heavy object in a similar fashion?

Examine the shapes below. Would logs of any of these shapes be able to roll heavy objects in a similar fashion? Be prepared to defend your conclusion!



7-3. Stan says that he has a tricycle with square wheels and claims that it can ride as smoothly as a tricycle with circular wheels! Rosita does not believe him. Analyze this possibility with your team as you answer the questions below.

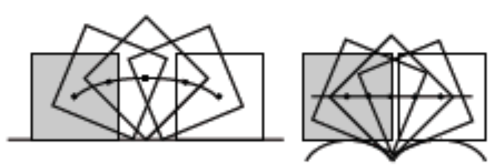


a. Is Stan's claim possible? Describe what it would be like to ride a tricycle with square tires. What type of motion would the rider experience? Why does this happen?

b. When Rosita challenged him, Stan confessed that he needed a special road so that the square wheels would be able to rotate smoothly and would keep Stan at a constant height. What would his road need to look like? Draw an example on your paper.

c. How would Stanley need to change his road to be able to ride a tricycle with rectangular (but non-square) wheels? Draw an example on your paper.

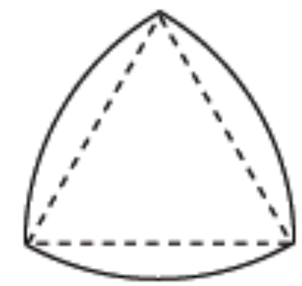
d. As the picture at right[1] shows, square wheels are possible if the road is specially curved to accommodate the change in the length of the radius of the wheel as it rotates. An example is shown below.



On a level surface *On a curved surface*

Using mathematical terms, explain why the square wheel needed a modified road to ride smoothly, while the circular wheel did not. What is different between the two shapes?

7-4. REULEAUX CURVES



Reuleaux (pronounced "roo LOW") curves are special because they have a constant diameter. That means that as a Reuleaux curve rotates, its height remains constant. Although the diagram at right is an example of a Reuleaux curve based on an equilateral triangle, these special curves can be based on any regular polygon with an odd number of sides.

a. What happens to the center (point *C*) as the Reuleaux wheel at right rolls?

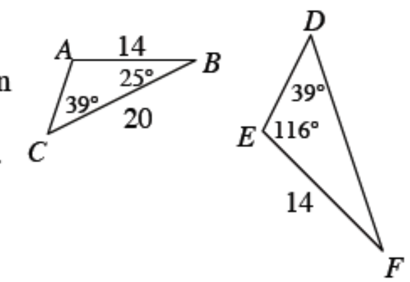
b. Since logs with a Reuleaux curve shape can also smoothly roll heavy objects, why are these shapes not used for bicycle wheels? In other words, what is the difference between a circle and a Reuleaux curve?

7-5. LEARNING LOG

A big focus of Chapters 7 through 12 is on circles. What did you learn about circles today? Did you learn anything about other shapes that was new or that surprised you? Write a Learning Log entry explaining what you learned about the shapes of wheels. Title this entry "Shapes of Wheels" and label it with today's date.

Review & Preview

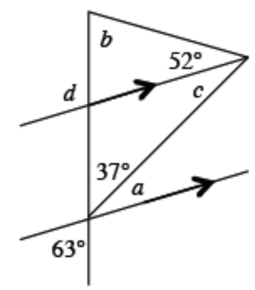
7-6. Examine ΔABC and ΔDEF at right.



a. Assume the triangles at right are not drawn to scale. Complete a flowchart to justify the relationship between the two triangles.

b. Find AC and DF.

7-7. Use the relationships in the diagram at right to find the values of each variable. Name which geometric relationships you used.

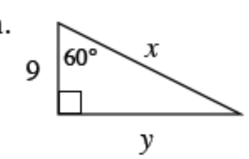


7-8. A rectangle has one side of length 11 mm and a diagonal of 61 mm. Draw a diagram of this rectangle and find its width and area.

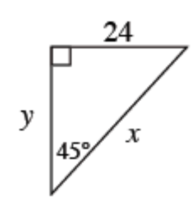
7-9. Troy is thinking of a shape. He says that it has four sides and that no sides have equal length. He also says that no sides are parallel. What is the best name for his shape?

7-10. Without using your calculator, find the exact values of x and y in each diagram below.

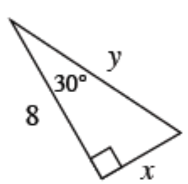
a.



b.



c.

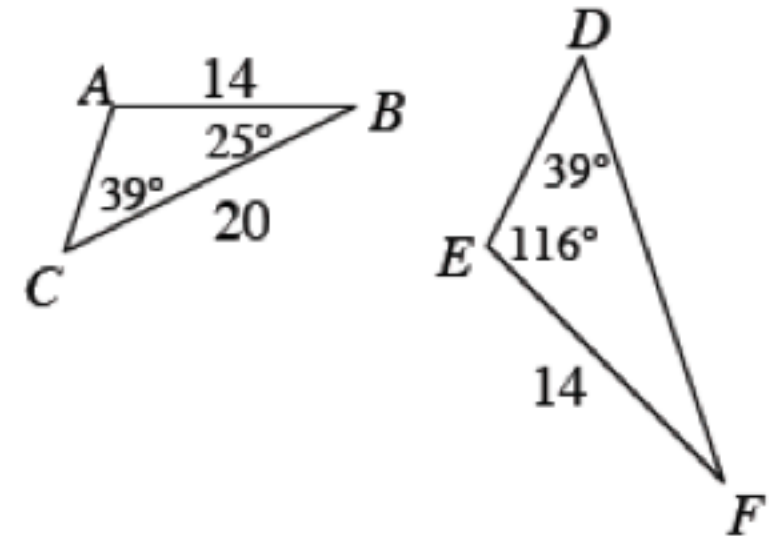


7-11. Solve each system of equations below, if possible. If it is not possible, explain what the lack of an algebraic solution tells you about the graphs of the equations. Write each solution in the form (x, y). Show all work.

a. $y = -2x - 1$
$y = \frac{1}{2}x - 16$

b. $y = x^2 + 1$
$y = -x^2$

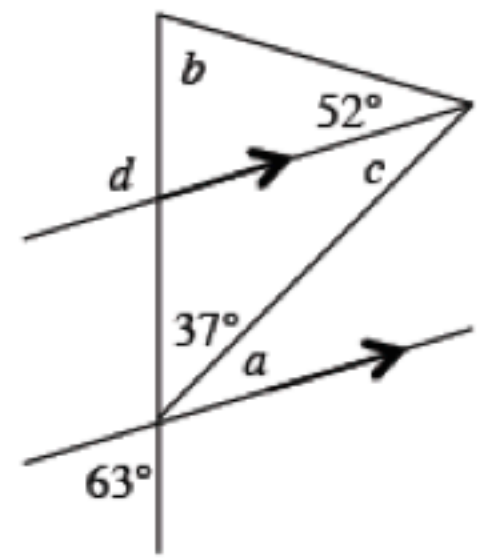
7-6. Examine ΔABC and ΔDEF at right.



a. Assume the triangles at right are not drawn to scale. Complete a flowchart to justify the relationship between the two triangles.

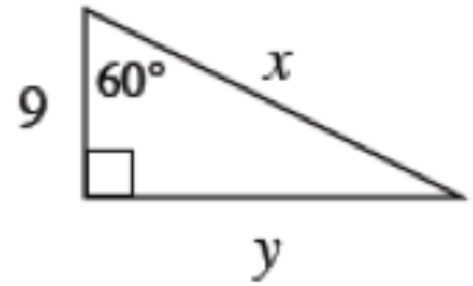
b. Find AC and DF.

7-7. Use the relationships in the diagram at right to find the values of each variable. Name which geometric relationships you used.

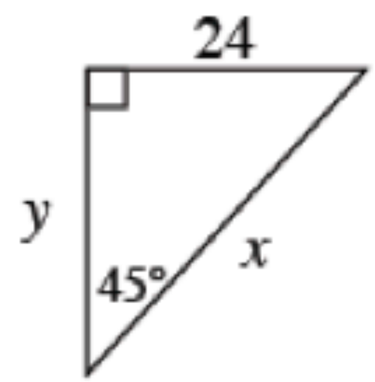


7-10. Without using your calculator, find the exact values of x and y in each diagram below.

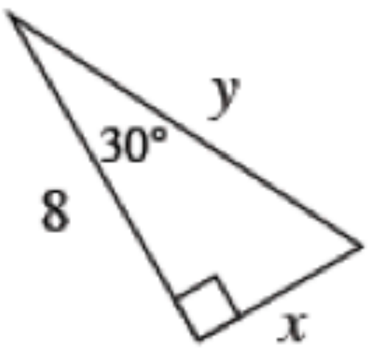
a.



b.

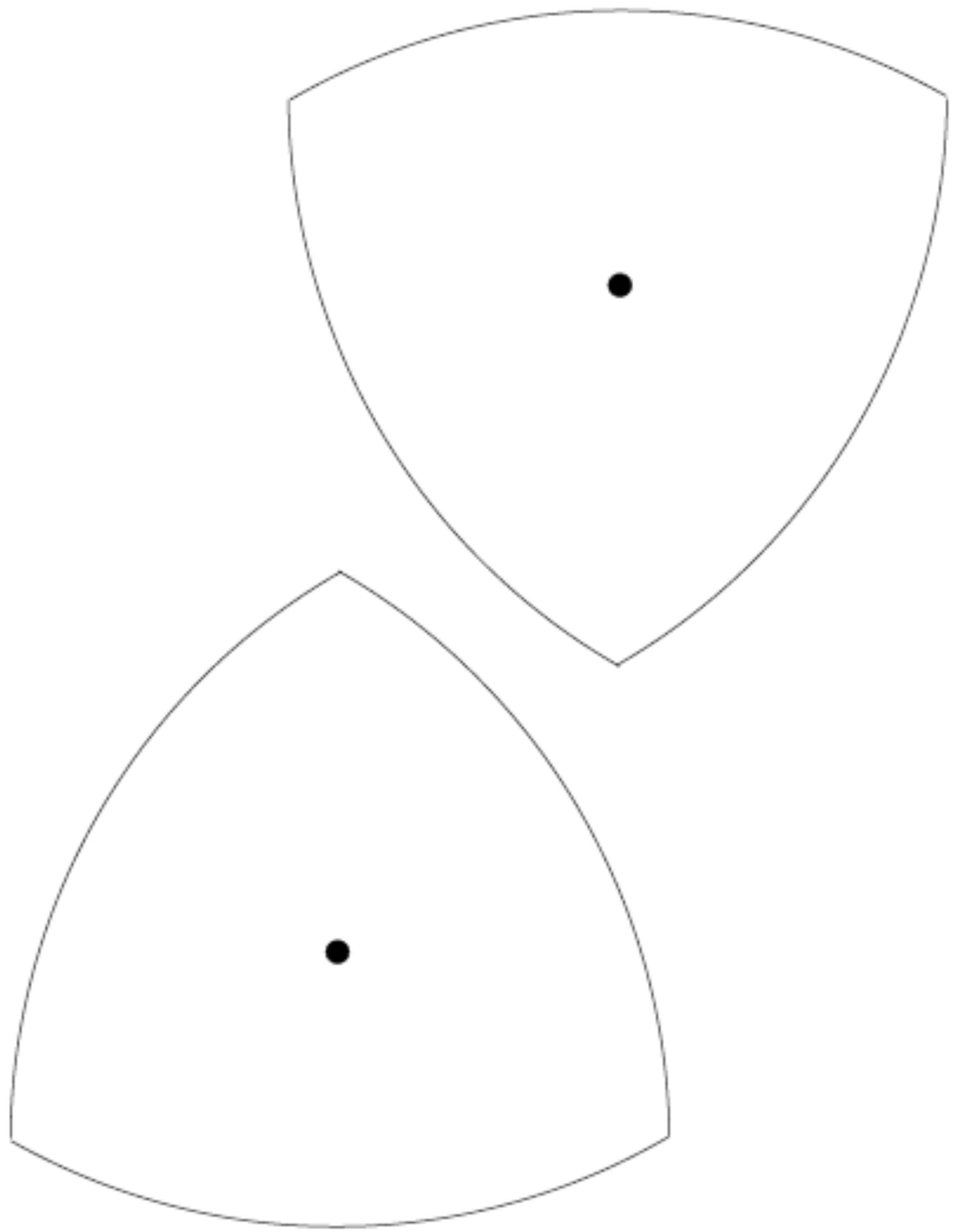


c.



Lesson 7.1.1A Resource Page

Reuleaux Wheels



Team Roles

Resource Manager:	• Get all materials for your team. • Make sure all your questions are team questions. Do not let your team stay stuck! *"Does anyone have any ideas?"* *"What team question can we ask the teacher?"* *"Are we sure no one here can answer the question?"*
Facilitator:	• Help your team get started by having someone read the task. Then make sure everyone understands what to do. *"Who wants to read?"* *"I'm not sure how to start – what are we being asked to do?"* *"What does the first question mean?"* • Make sure everyone's ideas are heard. *"Does anyone see it a different way?"* *"Does anyone have a different idea?"* • Keep your team together. Make sure everyone understands your team's conjectures and conclusions before moving on. *"Can you show us what you're doing?"* *"Do we all agree?"* *"Are we ready to move on?"* *"I'm not sure I get it yet – can someone explain?"*
Recorder/ Reporter:	• Your team needs to have a comprehensive list of brainstormed conjectures, tests performed, and counterexamples found with reasons. You will report on your team's process and results at the end of class. • As you work, it is your job to ensure your teammates can see each other's work, reasons, and connections. You might want to sketch information on scratch paper to put in the middle of your table so your team can talk about it. *"How can we show that idea?"* *"Should I make a picture for the middle of the table?"* *"How can we record that idea?"* *"Can you explain that idea again?"*
Task Manager:	• You need to make sure your team is accomplishing the task effectively and efficiently. Make sure that all talking is <u>within</u> your team and is helping you to accomplish the task. Eliminate side conversations. *"Okay, let's get back to work."* *"What does the next question say?"* • Listen for statements and reasons. *"Explain how you know that."* *"Can we prove that?"* and *"Tell why!"*

7.1.2 What can I build with a circle?

Building a Tetrahedron

In later chapters, you will learn more about polygons, circles, and three-dimensional shapes. Later investigations will require that you remember key concepts you have already learned about triangles, parallel lines, and other angle relationships. Today you will have the opportunity to review some of the geometry you have learned while also beginning to think about what you will be studying in the future.

As you work with your team, consider the following focus questions:

Is there more than one way?

How can you be sure that is true?

What else can we try?

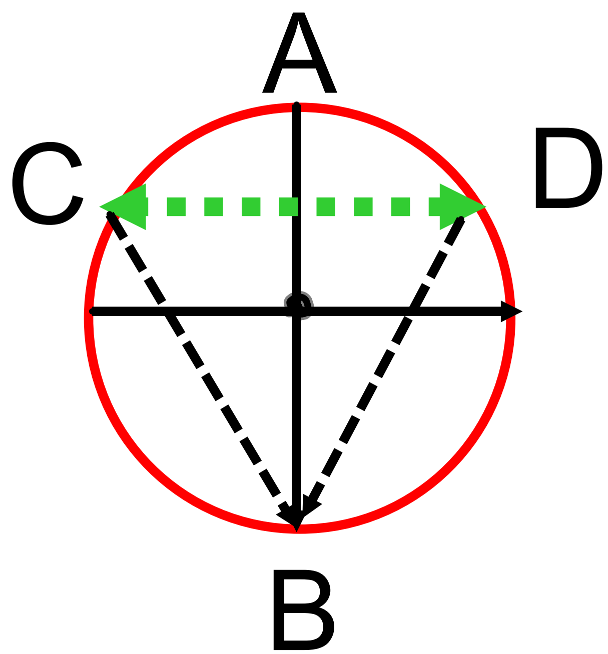
7-12. IS THERE MORE TO THIS CIRCLE?

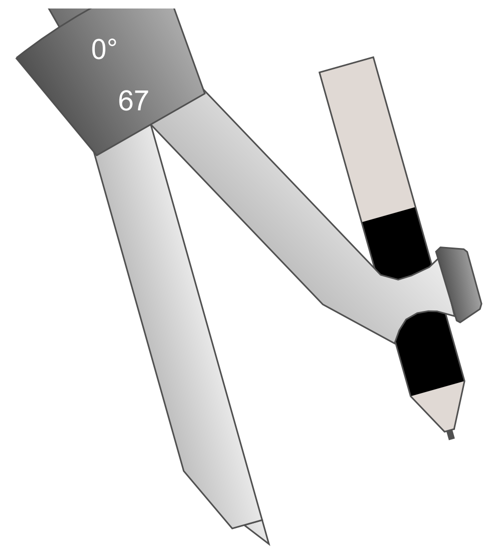
Circles can be folded to create many different shapes. Today, you will work with a circle and use properties of other shapes to develop a three-dimensional shape. Be sure to have reasons for each conclusion you make as you work. Each person in your team should start by obtaining a copy of a circle from your teacher and cutting it out.

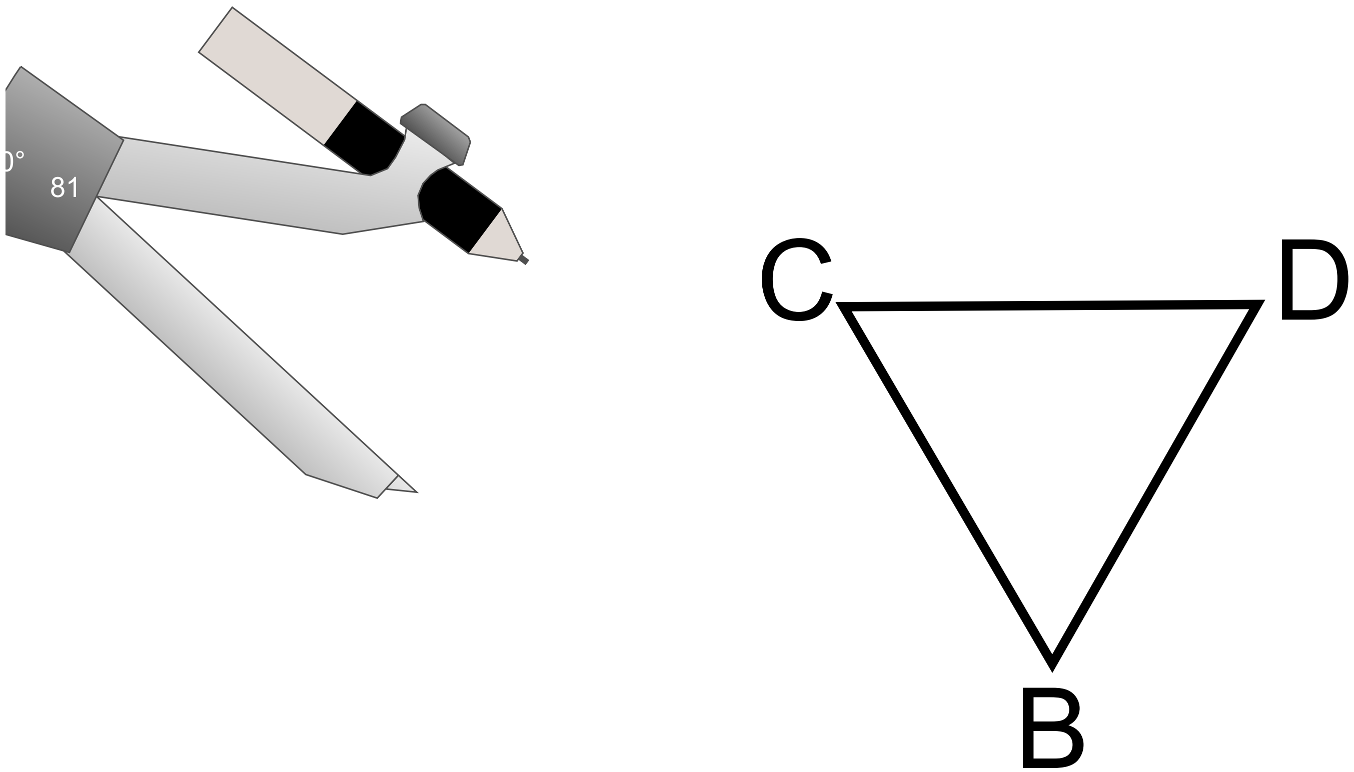
a. Fold the circle in half to create a crease that lies on a line of symmetry of the circle. Unfold the circle and then fold it in half again to create a new crease that is perpendicular to the first crease. Unfold your paper back to the full circle. How could you convince someone else that your creases are perpendicular? What is another name for the line segment represented by each crease?

b. On the circle, label the endpoints of one diameter *A* and *B*. Fold the circle so that point *A* touches the center of the circle and create a new crease. Then label the endpoints of this crease *C* and *D*. What appears to be the relationship between $\overline{AB}$ and $\overline{CD}$? Discuss and justify with your team. Be ready to share your reasons with the class.

c. Now fold the circle twice to form creases $\overline{BC}$ and $\overline{BD}$ and use scissors to cut out ΔBCD. What type of triangle is ΔBCD? How can you be sure? Be ready to convince the class.


A
C
D
B


0°
67


C
D
B

7-13. ADDING DEPTH

Your equilateral triangle should now be flat (also called two-dimensional). **Two-dimensional** shapes have length and width, but not depth (or "thickness").

a. If the labels were cut off, then label the vertices of ΔBCD again now. Then, with the unmarked side of the triangle facedown, fold and crease the triangle so that B touches the midpoint of $\overline{CD}$. Keep it in the folded position.

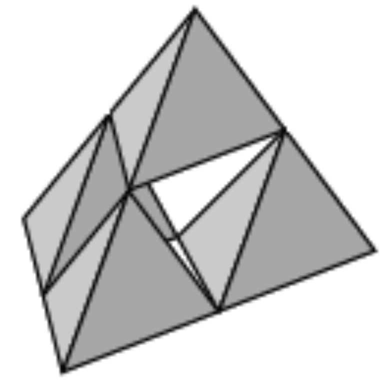
What does the resulting shape appear to be? What smaller shapes do you see inside the larger shape? Justify that your ideas are correct. For example, if you think that lines are parallel, you must provide evidence.

b. Open your shape again so that you have the large equilateral triangle in front of you. How does the length of a side of the large triangle compare to the length of the side of the small triangle formed by the crease? How many of the small triangles would fit inside the large triangle? In what ways are the small and large triangles related?

c. Repeat the fold in part (a) so that C touches the midpoint of $\overline{BD}$. Unfold the triangle and fold again so that D touches the midpoint of $\overline{BC}$. Create a three-dimensional shape by bringing points B, C, and D together. A **three-dimensional** shape has length, width, and depth. Use tape to hold your shape together.

d. Three-dimensional shapes formed with polygons have **faces** and **edges**, as well as **vertices** (plural of vertex). Faces are the flat surfaces of the shape, while edges are the line segments formed when two faces meet. Vertices are the points where edges intersect. Discuss with your team how to use these words to describe your new shape. Then write a complete description. If you think you know the name of this shape, include it in your description.

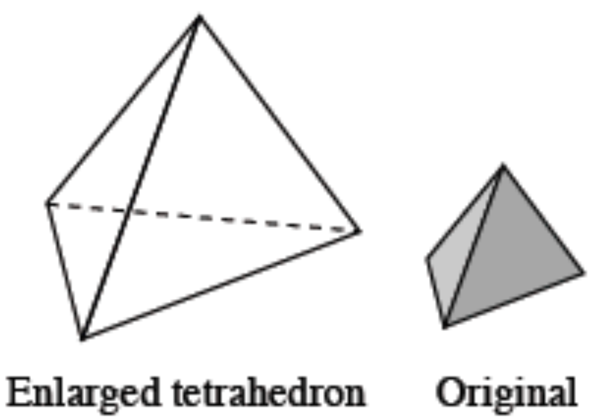
7-14. Your team should now have 4 three-dimensional shapes (called tetrahedra). If you are working in a smaller team, you should quickly fold more shapes so that you have a total of four.



a. Put four tetrahedra together to make an enlarged tetrahedron like the one pictured at right. Is the larger tetrahedron similar to the small tetrahedron? How can you tell?

b. To determine the edges and faces of the new shape, pretend that it is solid. How many edges does a tetrahedron have? Are all of the edges the same length? How does the length of an edge of the team shape compare with the length of an edge of one of the small shapes?

c. How many faces of the small tetrahedral would it take to cover the face of the large tetrahedron? Remember to count gaps as part of a face. Does the area of the tetrahedron change in the same way as the length?

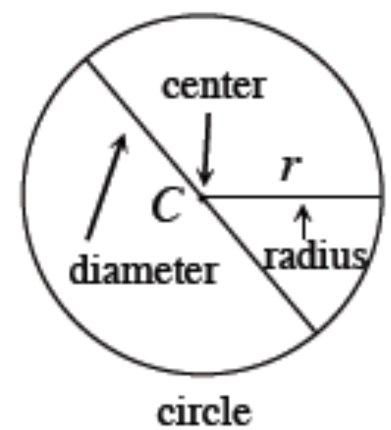


Methods and Meanings

MATH NOTES

Parts of a Circle

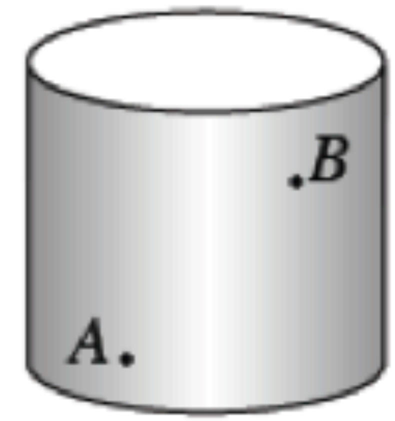
A **circle** is the set of all points on a flat surface that are the same distance from a fixed central point, C, referred to as its **center**. This text will use the notation $\odot C$ to name a circle with center at point C.



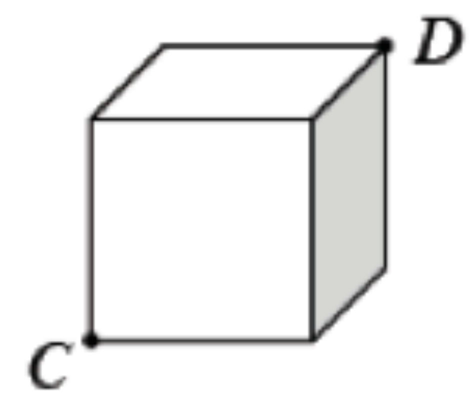
The **radius** is a line segment from the center to a point on the circle. Its length is usually denoted r. However, a line segment drawn through the center of the circle with both endpoints on the circle is called a **diameter** and its length is usually denoted d.

Notice that a diameter of a circle is always twice as long as the radius.

a. In this first puzzle, Bradley decided to test what would happen on the side of a cylinder, such as a soup can. On a can provided by your teacher, find points *A* and *B* labeled on the outside of the can. With your team, determine the shortest path from point *A* to point *B* along the surface of the can. (In other words, no part of your path can go inside the can.) Describe how you found your solution.

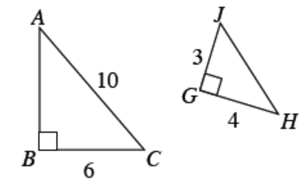


b. What if the shape is a cube? Using a cube provided by your teacher, predict which path would be the shortest path from opposite corners of the cube (labeled points *C* and *D* in the diagram at right). Then test your prediction. Describe how you found the shortest path.

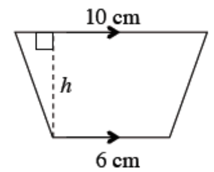


Review & Preview

7-15. What is the relationship between ΔABC and ΔGHJ at right? Create a flowchart to justify your conclusion.



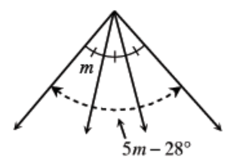
7-16. The area of the trapezoid at right is 56 cm². What is h? Show all work.



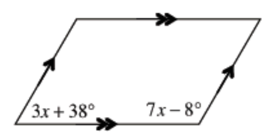
7-17. Line L is perpendicular to the line $6x - y = 7$ and passes through the point (0, 6). Line M is parallel to the line $y = \frac{2}{3}x - 4$ and passes through the point (–3, –1). Where do these lines intersect? Explain how you found your solution.

7-18. Examine the geometric relationships in each of the diagrams below. For each one, write and solve an equation to find the value of the variable. Name any geometric property or conjecture that you used.

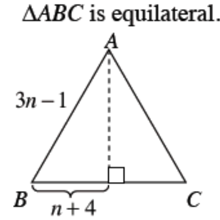
a.



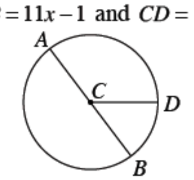
b.



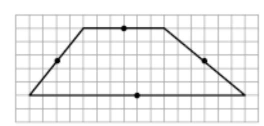
c. ΔABC is equilateral.



d. Point C is the center of the circle. $AB = 11x - 1$ and $CD = 3x + 12$



7-19. In the Shape Factory, you created many shapes by rotating triangles about the midpoint of its sides. (Remember that the **midpoint** is the point exactly halfway between the endpoints of the line segment.) However, what if you rotate a trapezoid instead?



Carefully draw the trapezoid above on graph paper, along with the given midpoints. Then rotate the trapezoid 180° about one of the midpoints and examine the resulting shape formed by both trapezoids (the original and its image). Continue this process with each of the other midpoints, until you discover all the shapes that can be formed by a trapezoid and its image when rotated 180° about the midpoint of one of its sides.

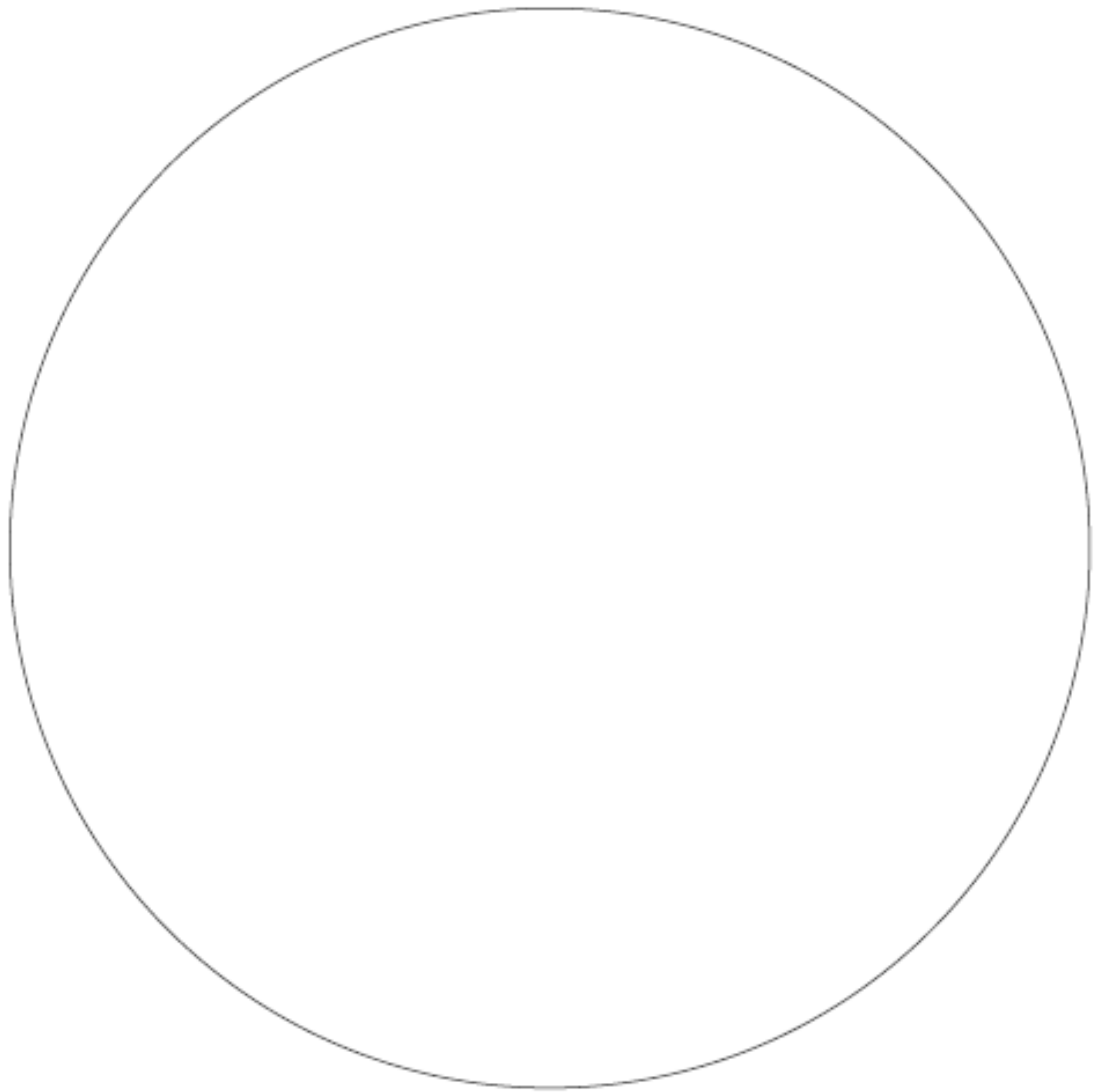
7-20. On graph paper, plot the points $A(-5, 7)$ and $B(3, 1)$.

a. Find AB (the length of $\overline{AB}$).

b. Locate the midpoint of $\overline{AB}$ and label it C. What are the coordinates of C?

c. Find AC. Can you do this without using the Pythagorean Theorem?

Circle



General Team Roles

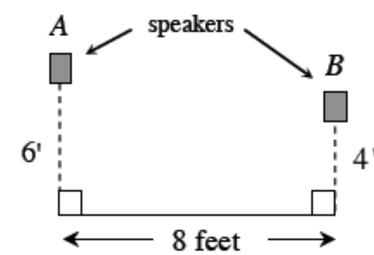
	Resource Manager: • Get supplies for your team, and make sure your team cleans up. • Call the teacher over for team questions: *"No one has an idea? Should I call the teacher?"*
	Facilitator: • Help your team get started by having someone read the task: *"Who wants to read?"* • Make sure everyone understands what to do: *"Does anyone know how to get started?"* *"What does the first question mean?"* *"I'm not sure – what are we supposed to do?"* • Make sure everyone understands your team's answer before you move on: *"Do we all agree?"* *"I'm not sure I get it yet – can someone explain?"*
	Recorder/Reporter: • Share your team data with the class. • Be sure all team members have access to any team diagrams by placing them at the center of the table or desks. • Make sure your team agrees about how to show your work: *"How can we write this?"* *"How can we show it on the diagram?"*
	Task Manager: • Make sure no one talks outside your team. • Help keep your team on-task and talking about math: *"Okay, let's get back to work!"* *"Let's keep working."* • Listen for statements and reasons: *"Explain how you know that."* *"Can you prove that?"*

7.1.3 What is the shortest distance?

Shortest Distance Problems

Questions such as, “What length will result in the largest area?” or “When was the car traveling the slowest?” concern *optimization*. To optimize a quantity is to find the “best” possibility. Calculus is often used to solve optimization problems, but geometric tools can sometimes offer surprisingly simple and elegant solutions.

7-21. INTERIOR DESIGN



Laura needs your help. She needs to order expensive wire to connect her sound system to her built-in speakers and would like your help to save her money.

She plans to place her sound system somewhere on a cabinet that is 8 feet wide. Speaker A is located 6 feet above one end of the cabinet, while speaker B is located 4 feet above the other end. She will need wire to connect the sound system to speaker A, and additional wire to connect it to speaker B.

Where should she place her sound system that she needs the least amount of wire?

Your Task: Before you discuss this with your team, make your own guess. What does your intuition tell you? Then, using the Lesson 7.1.3 Resource Page, work with your team to determine where on the cabinet the sound system should be placed. How can you be sure that you found the best answer? In other words, how do you know that the amount of wire you found is the least amount possible?

Discussion Points

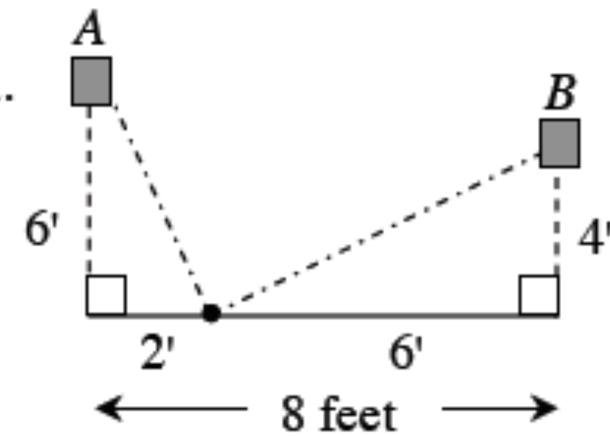
What is this problem about? What are you supposed to find?

What is a reasonable estimate of the total length of speaker wire?

What mathematical tools could be helpful to solve this problem?

Further Guidance

7-22. To help solve problem 7-21, first collect some data.



a. Calculate the total length of wire needed if the sound system is placed 2 feet from the left edge of the cabinet (the edge below Speaker A), as shown in the diagram at right.

b. Now calculate the total length of wire needed if the sound system is placed 3 feet from the same edge. Does this placement require more or less wire than that from part (a)?

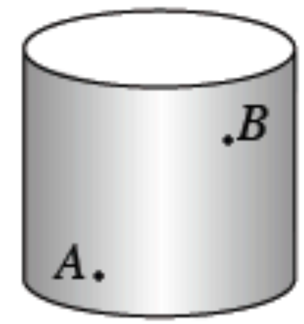
c. Continue testing placements for the sound system and create a table with your results. Where should the sound system be placed to minimize the amount of wire?

Further Guidance section ends here.

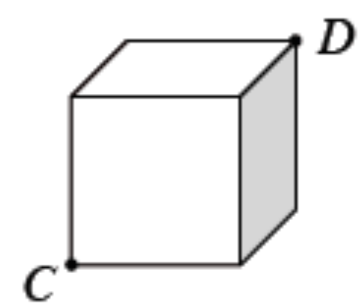
7-23. This problem reminds Bradley of problem 3-105, *You Are Getting Sleepy…*, in which you and a partner created two triangles by standing and gazing into a mirror. He remembered that the only way two people could see each other's eyes in the mirror was when the triangles were similar. Examine your solution to problem 7-21. Are the two triangles created by the speaker wires similar? Justify your conclusion.

7-24. Bradley enjoyed solving problem 7-21 so much that he decided to create other "shortest distance" problems. For each situation below, first predict where the shortest path would be using visualization and intuition. Then find a way to determine whether the path you chose is, in fact, the shortest.

a. In this first puzzle, Bradley decided to test what would happen on the side of a cylinder, such as a soup can. On a can provided by your teacher, find points *A* and *B* labeled on the outside of the can. With your team, determine the shortest path from point *A* to point *B* along the surface of the can. (In other words, no part of your path can go inside the can.) Describe how you found your solution.



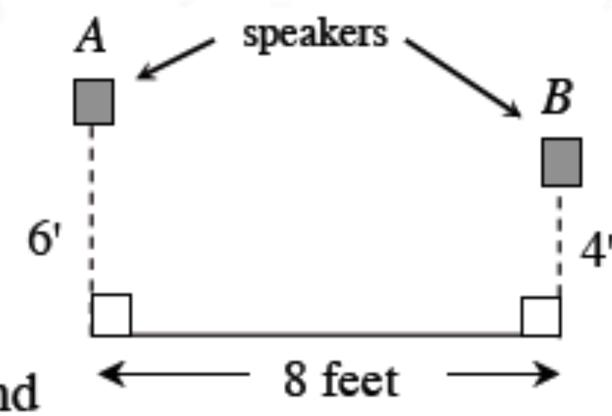
b. What if the shape is a cube? Using a cube provided by your teacher, predict which path would be the shortest path from opposite corners of the cube (labeled points *C* and *D* in the diagram at right). Then test your prediction. Describe how you found the shortest path.



7-25. MAKING CONNECTIONS

As Bradley looked over his answer from problem 7-21, he couldn't help but wonder if there is a way to change this problem into a straight-line problem like those in problem 7-24.

a. On the Lesson 7.1.3 Resource Page, reflect one of the speakers so that when the two speakers are connected with a straight line, the line passes through the horizontal cabinet.



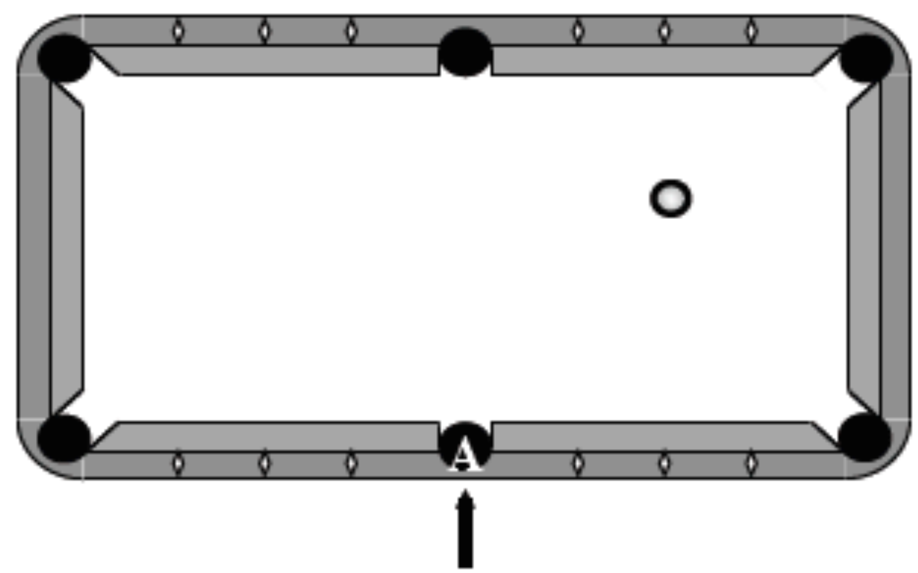
b. When the speakers from part (a) are connected with a straight line, two triangles are formed. How are the two triangles related? Justify your conclusion.

c. Use the fact that the triangles are similar to find where the sound system should be placed. Did your answer match that from problem 7-21?

7-26. TAKE THE SHOT

While playing a game of pool, Montana Mike needed to hit the last remaining ball into pocket A, as shown in the diagram below. However, to show off, he decided to make the ball first hit at least one of the rails of the table.

Your Task: On the Lesson 7.1.3 Resource Page provided by your teacher, determine where Mike could bounce the ball off a rail so that it will land in pocket A. Work with your team to find as many possible locations as you can. Can you find a way he could hit the ball so that it would rebound twice before entering pocket A?



Be ready to share your solutions with the class.

7-27. LEARNING LOG

Look over your work from this lesson. What mathematical ideas did you use? What connections, if any, did you find? Can any other problems you have seen so far be solved using a straight line? Describe the mathematical ideas you developed during this lesson in your Learning Log. Title this entry "Shortest Distance" and label it with today's date.

MATH NOTES

METHODS AND MEANINGS

Congruent Triangles → Congruent Corresponding Parts

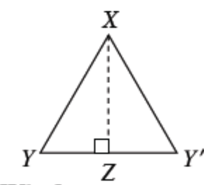
As you learned in Chapter 3, if two shapes are congruent, then they have exactly the same shape and the same size. This means that if you know two triangles are congruent, you can state that corresponding parts are congruent. This can be also stated with the arrow diagram:

$$\cong \Delta\text{s} \rightarrow \cong \text{parts}$$

For example, if $\Delta ABC \cong \Delta PQR$, then it follows that $\angle A \cong \angle P$, $\angle B \cong \angle Q$, and $\angle C \cong \angle R$. Also, $\overline{AB} \cong \overline{PQ}$, $\overline{AC} \cong \overline{PR}$, and $\overline{BC} \cong \overline{QR}$.

Review & Preview

7-28. ΔXYZ is reflected across $\overline{XZ}$, as shown at right.



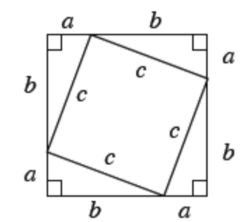
a. How can you justify that the points Y, Z, and Y' all lie on a straight line?

b. What is the relationship between ΔXYZ and $\Delta X'Y'Z'$? Why?

c. Read the Math Notes box for this lesson. Then make all the statements you can about the corresponding parts of these two triangles.

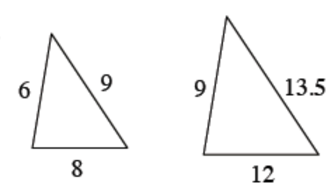
7-29. Remember that a midpoint of a line segment is the point that divides the segment into two segments of equal length. On graph paper, plot the points $P(0, 3)$ and $Q(0, 11)$. Where is the midpoint M if $PM = MQ$? Explain how you found your answer.

7-30. Examine the diagram at right. Find two equivalent expressions that represent the area of the *inner* square.

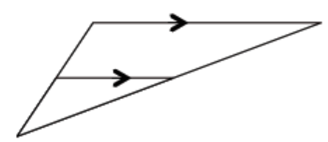


7-31. For each pair of triangles below, decide whether the triangles are similar and/or congruent. Justify each conclusion.

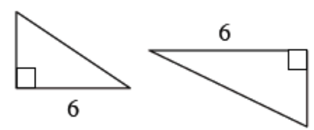
a.



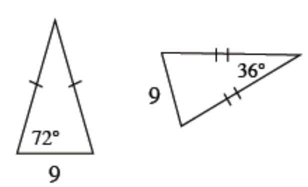
b.



c.



d.



7-32. On graph paper, plot and connect the points $A(1, 1)$, $B(2, 3)$, $C(5, 3)$, and $D(4, 1)$ to form quadrilateral $ABCD$.

a. What is the best name for quadrilateral $ABCD$? Justify your answer.

b. Find and compare $m\angle DAB$ and $m\angle BCD$. What is their relationship?

c. Find the equations of diagonals $\overline{AC}$ and $\overline{BD}$. Are the diagonals perpendicular?

d. Find the point where diagonals $\overline{AC}$ and $\overline{BD}$ intersect.

7-33. Solve each system of equations below, if possible. If it is not possible, explain what having "no solution" tells you about the graphs of the equations. Write each solution in the form (x, y). Show all work.

a. $y = -\frac{1}{3}x + 7$
$y = -\frac{1}{3}x - 2$

b. $y = 2x + 3$
$y = x^2 - 2x + 3$

7-34. How long is the longest line segment that will fit inside a square of area 50 square units? Show all work.

7-35. Graph and connect the points $G(-2, 2)$, $H(3, 2)$, $I(6, 6)$, and $J(1, 6)$ to form $GHIJ$.

a. What specific type of shape is quadrilateral $GHIJ$? Justify your conclusion.

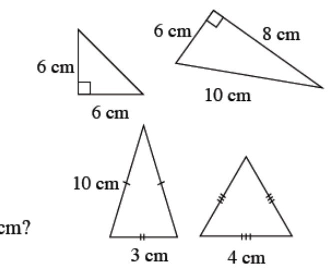
b. Find the equations of the diagonals $\overline{GI}$ and $\overline{HJ}$.

c. Compare the slopes of the diagonals. How do the diagonals of a rhombus appear to be related?

d. Find J' if quadrilateral $GHIJ$ is rotated 90° clockwise (↻) about the origin.

e. Find the area of quadrilateral $GHIJ$.

7-36. The four triangles at right are placed in a bag. If you reach into the bag without looking and pull out one triangle at random, what is the probability that:



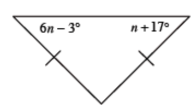
a. The triangle is scalene?

b. The triangle is isosceles?

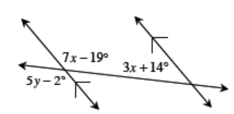
c. at least one side of the triangle is 6 cm?

7-37. Examine the relationships in the diagrams below. For each one, write an equation and solve for the given variable(s). Show all work.

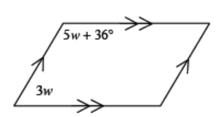
a.



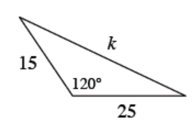
b.



c.



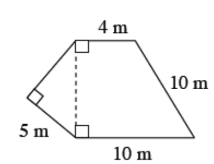
d.



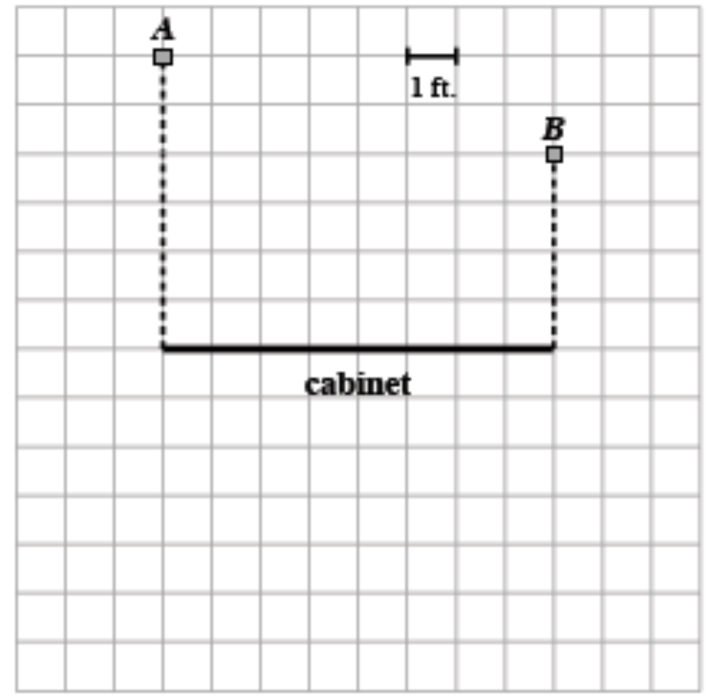
7-38. Copy the table below on your paper and complete it for the equation $y = x^2 + 2x - 3$. Then graph and connect the points on graph paper. Name the roots (x-intercepts).

x	–4	–3	–2	–1	0	1	2
y							

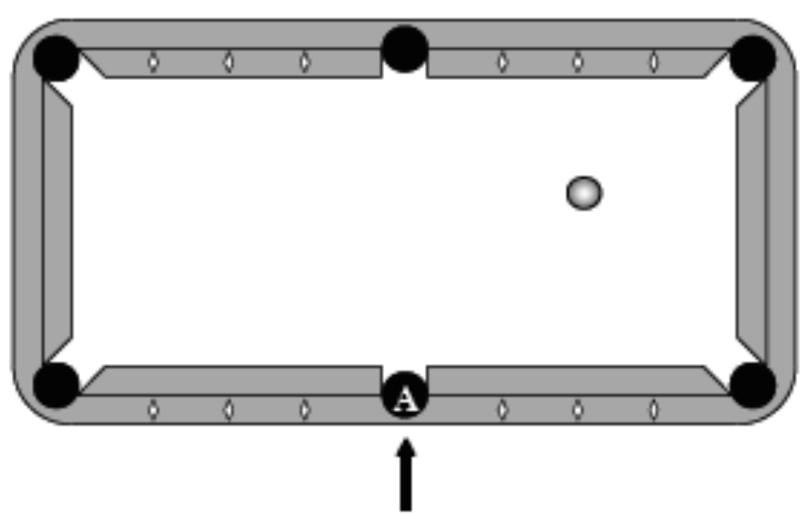
7-39. Find the perimeter of the shape at right. Show all work.



7-21. INTERIOR DESIGN



7-26. TAKE THE SHOT



7.1.4 How can I create it?

Using Symmetry to Study Polygons

In Chapter 1, you used a hinged mirror to study the special angles associated with regular polygons. In particular, you investigated what happens as the angle formed by the sides of the mirror is changed. Toda y, you will use a hinged mirror to determine if there is more than one way to build each regular polygon using the principles of symmetry. And what about other types of polygons? What can a hinged mirror help you understand about them?

As your work with your study team, keep these focus questions in mind:

Is there another way?

What types of symmetry can I find?

What does symmetry tell me about the polygon?

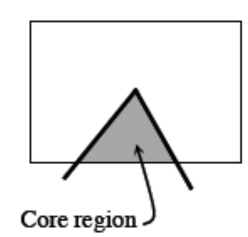
7-40. THE HINGED MIRROR TEAM CHALLENGE

Obtain a hinged mirror, a piece of unlined colored paper, and a protractor from your teacher.

With your team, spend five minutes reviewing how to use the mirror to create regular polygons. Remember that a **regular polygon** has equal sides and angles. Once everyone remembers how the hinged mirror works, select a team member to read the directions of the task below.

Your Task: Below are four challenges for your team. Each requires you to find a creative way to position the mirror in relation to the colored paper. You can tackle the challenges in any order, but you must work together as a team on each of them. Whenever you successfully create a shape, do not forget to measure the angle formed by the mirror, as well as draw a diagram on your paper of the core region in front of the mirror. If your team decides that a shape is impossible to create with the hinged mirror, explain why.

- Create a regular hexagon.
- Create an equilateral triangle at least *two* different ways.
- Create a rhombus that is *not* a square.
- Create a circle.



7-41. ANALYSIS

How can symmetry help you to learn more about shapes? Discuss each question below with the class.

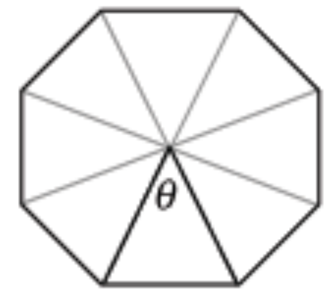
a. One way to create a regular hexagon with a hinged mirror is with six triangles, as shown in the diagram at right. Note: The gray lines represent reflections of the bottom edges of the mirrors and the edge of the paper, while the core region is shaded.

What is special about each of the triangles in the diagram? What is the relationship between the triangles? Support your conclusions. Would it be possible to create a regular hexagon with 12 triangles? Explain.

b. If you have not done so already, create an equilateral triangle so that the core region in front of the mirror is a right triangle. Draw a diagram of the result that shows the different reflected triangles like the one above. What special type of right triangle is the core region? Can all regular polygons be created with a right triangle in a similar fashion?

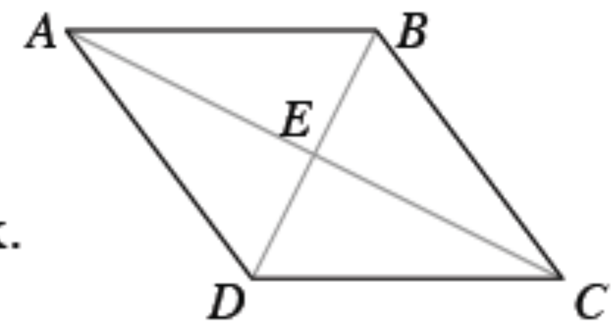
c. In problem 7-40, your team formed a rhombus that is not a square. On your paper, draw a diagram like the one above that shows how you did it. How can you be sure your resulting shape is a rhombus? Using what you know about the angle of the mirror, explain what must be true about the diagonals of a rhombus.

7-42. Use what you learned today to answer the questions below.



a. Examine the regular octagon at right. What is the measure of angle θ? Explain how you know.

b. Quadrilateral *ABCD* at right is a rhombus. If $BD = 10$ units and $AC = 18$ units, then what is the perimeter of *ABCD*? Show all work.



MATH NOTES

METHODS AND MEANINGS

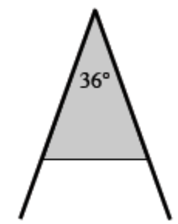
Regular Polygons

A polygon is **regular** if all its sides are congruent and its angles have equal measure. An equilateral triangle and a square are each regular polygons since they are both *equilateral* and *equiangular*. See the diagrams of common regular polygons below.

Equilateral Triangle | Square | Regular Hexagon | Regular Octagon | Regular Decagon

Review & Preview

7-43. Felipe set his hinged mirror so that its angle was 36° and the core region was isosceles, as shown at right.



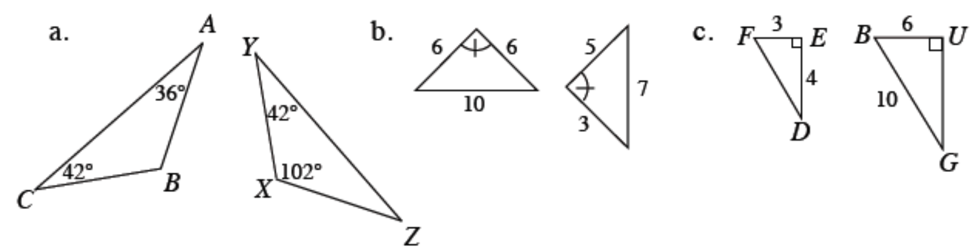
a. How many sides did his resulting polygon have? Show how you know.

b. What is another name for this polygon?

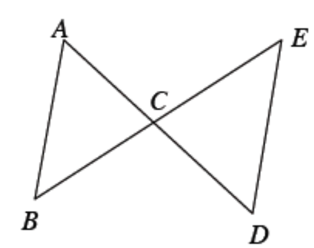
7-44. In problem 7-41 you learned that the diagonals of a rhombus are perpendicular bisectors. If *ABCD* is a rhombus with side length 15 mm and if $BD = 24$ mm, then find the length of the other diagonal, $\overline{AC}$. Draw a diagram and show all work.

7-45. Joanne claims that (2, 4) is the midpoint of the segment connecting the points (–3, 5) and (7, 3). Is she correct? Explain how you know.

7-46. For each pair of triangles below, determine whether or not the triangles are similar. If they are similar, show your reasoning in a flowchart. If they are not similar, explain how you know.



7-47. If $\Delta ABC \cong \Delta DEC$, which of the statements below must be true? Justify your conclusion. Note: More than one statement may be true.



a. $\overline{AC} \cong \overline{DC}$ b. $m\angle B = m\angle D$

c. $\overline{AB} \parallel \overline{DE}$ d. $AD = BE$

e. None of these are true.

7-48. On graph paper, graph the points *A*(2, 9), *B*(4, 3), and *C*(9, 6). Which point (*A* or *C*) is closer to point *B*? Justify your conclusion.