Agenda: #59F AP Stats Ch18C + Quiz Review

Title: Choosing Your Sample Size

Objectives: According to the given Margin of Error and z*, we calculate choosing sample size.

1) W.Up : (15min) Use yesterday's worksheet and fine "one-proportion-z-interval".

2) Global Warming Questions

(Reading+ Activity 20 min)

3) Credit Card Questions

(Elbow Partner 15min)

4) Japanese Cars Questions (15min)

5) Worksheet 2nd Page 1~5

(Independent 20min)

6) Closing Activity (10min)

Each confidence interval discussed in the book has a name. You'll see many different kinds of confidence intervals in the following chapters. Some will be about more than *one* sample, some will be about statistics other than *proportions,* and some will use models other than the Normal. The interval calculated and interpreted here is sometimes called a **one-proportion z-interval.**[4]

The extent of the interval on either side of $\hat{p}$ is called the **margin of error** (*ME*).

We'll want to use the same approach for many other situations besides estimating proportions. In fact, almost any population parameter—a proportion, a mean, or a regression slope, for example—can be estimated with some margin of error. The margin of error is a way to describe our uncertainty in estimating the population value. We'll see how to find a margin of error for each of these values and for others.

For all of those statistics, regardless of how we calculate the margin of error, we'll be able to construct a confidence interval that looks like this:

$$Estimate \pm ME.$$

The margin of error for our 95% confidence interval was 2 *SE*. What if we wanted to be more confident? To be more confident, we'll need to capture p more often, and to do that we'll need to make the interval wider. For example, if we want to be 99.7% confident, the margin of error will have to be 3 *SE*.

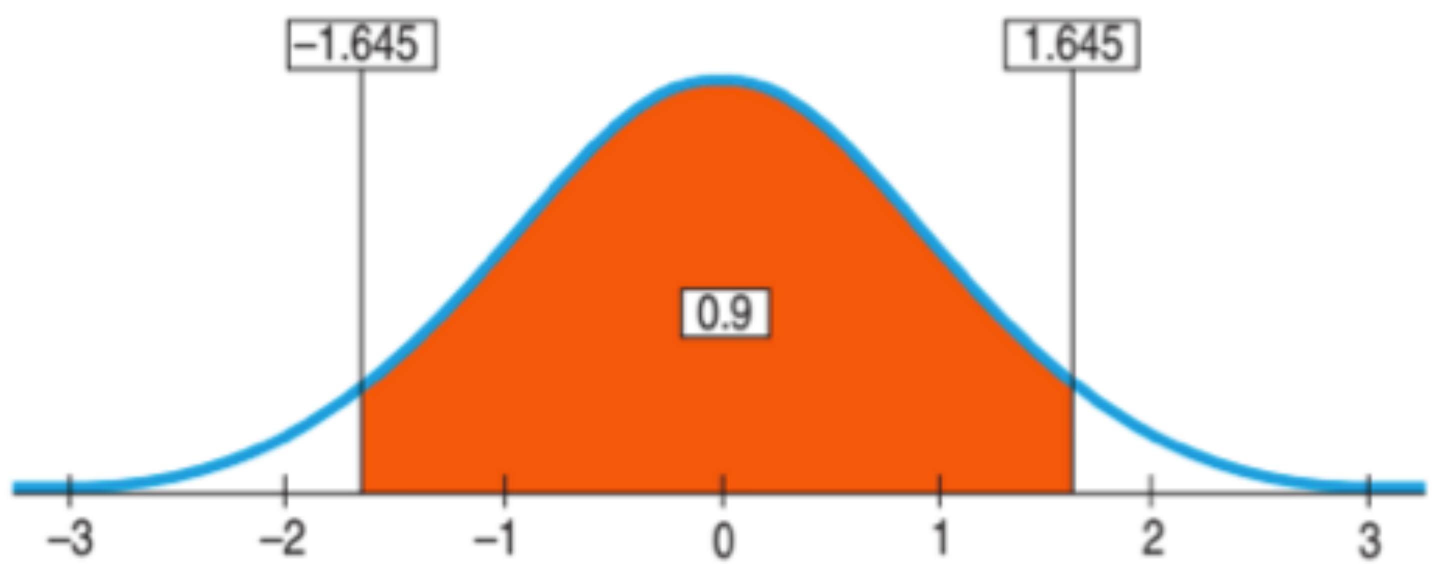
Critical Values

NOTATION ALERT

We'll put an asterisk on a letter to indicate a critical value, so z^* is always a critical value from a normal model.

In our sea fans example we used $2SE$ to give us a 95% confidence interval. To change the confidence level, we'd need to change the *number* of SEs so that the size of the margin of error corresponds to the new level. This number of SEs is called the **critical value.** Here it's based on the Normal model, so we denote it z^*. For any confidence level, we can find the corresponding critical value from a computer, a calculator, or a Normal probability table, such as Table Z.

For a 95% confidence interval, you'll find the precise critical value is $z^* = 1.96$. That is, 95% of a Normal model is found within ± 1.96 standard deviations of the mean. We've been using $z^* = 2$ from the 68–95–99.7 Rule because it's easy to remember.



Independence Assumption

Independence Assumption: The data values must be independent. To think about whether this assumption is plausible, we often look for reasons to suspect that it fails. We wonder whether there is any reason to believe that the data values somehow affect each other. (For example, might the disease in sea fans be contagious?) Whether you decide that the **Independence Assumption** is plausible depends on your knowledge of the situation. It's not one you can check by looking at the data.

However, now that we have data, there are two conditions that we can check:

Randomization Condition: Were the data sampled at random or generated from a properly randomized experiment? Proper randomization can help ensure independence.

10% Condition: If you sample more than 10% of a population, the formula for the standard error won't be quite right. There is a special formula (found in advanced books) that corrects for this, but it isn't a common problem unless your population is small.

Sample Size Assumption

The model we use for inference for proportions is based on the Central Limit Theorem. We need to know whether the sample is large enough to make the sampling model for the sample proportions approximately Normal. It turns out that we need more data as the proportion gets closer and closer to either extreme (0 or 1). That's why we check the:

Success/Failure Condition: We must expect at least 10 "successes" and at least 10 "failures." Recall that by tradition we arbitrarily label one alternative (usually the outcome being counted) as a "success" even if it's something bad (like getting a disease). The other alternative is, of course, then a "failure."

One-Proportion *z*-Interval

When the conditions are met, we are ready to find a level C confidence interval for the population proportion, p. The confidence interval is $\hat{p} \pm z^* \times SE(\hat{p})$ where the standard deviation of the proportion is estimated by $SE(\hat{p}) = \sqrt{\frac{\hat{p}\hat{q}}{n}}$ and the critical value, z^*, specifies the number of SEs needed for C% of random samples to yield confidence intervals that capture the true parameter value.

Choosing Your Sample Size

The question of how large a sample to take is an important step in planning any study. We weren't ready to make that calculation when we first looked at study design in Chapter 11, but now we can—and we always should.

Suppose a candidate is planning a poll and wants to estimate voter support within 3% with 95% confidence. How large a sample does she need?

Let's look at the margin of error:

$$ME = z^*\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.03 = 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}}.$$

We want to find n, the sample size. To find n we need a value for $\hat{p}$. We don't know $\hat{p}$ because we don't have a sample yet, but we can probably guess a value. The worst case—the value that makes $\hat{p}\hat{q}$ (and therefore n) largest—is 0.50, so if we use that value for $\hat{p}$, we'll certainly be safe. Our candidate probably expects to be near 50% anyway.

Our equation, then, is

$$0.03 = 1.96\sqrt{\frac{(0.5)(0.5)}{n}}.$$

To solve for n, we first multiply both sides of the equation by $\sqrt{n}$ and then divide by 0.03:

$$0.03\sqrt{n} = 1.96\sqrt{(0.5)(0.5)}$$

$$\sqrt{n} = \frac{1.96\sqrt{(0.5)(0.5)}}{0.03} \approx 32.67$$

Notice that evaluating this expression tells us the *square root* of the sample size. We need to square that result to find n:

$$n \approx (32.67)^2 \approx 1067.1$$

To be safe, we round up and conclude that we need at least 1068 respondents to keep the margin of error as small as 3% with a confidence level of 95%.

For Example CHOOSING A SAMPLE SIZE

RECAP: The Yale/George Mason poll that estimated that 40% of all voters believed that scientists disagree about whether global warming exists had a margin of error of $\pm 3\%$. Suppose an environmental group planning a follow-up survey of voters' opinions on global warming wants to determine a 95% confidence interval with a margin of error of no more than $\pm 2\%$.

QUESTION: How large a sample do they need? (You could take $p = 0.5$, but we have data that indicate $p = 0.40$, so we can use that.)

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ANSWER:

$$ME = z^*\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.02 = 1.96\sqrt{\frac{(0.40)(0.60)}{n}}$$

$$\sqrt{n} = \frac{1.96\sqrt{(0.40)(0.60)}}{0.02} \approx 48.01$$

$$n = 48.01^2 = 2304.96$$

The environmental group's survey will need at least 2305 respondents.

Unfortunately, bigger samples cost more money and more effort. Because the standard error declines only with the *square root* of the sample size, to cut the standard error (and thus the ME) in half, we must *quadruple* the sample size.

Generally a margin of error of 5% or less is acceptable, but different circumstances call for different standards. For a pilot study, a margin of error of 10% may be fine, so a sample of 100 will do quite well. In a close election, a polling organization might want to get the margin of error down to 2%. Drawing a large sample to get a smaller ME, however, can run into trouble. It takes time to survey 2400 people, and a survey that extends over a week or more may be trying to hit a target that moves during the time of the survey. An important event can change public opinion in the middle of the survey process.

Keep in mind that the sample size for a survey is the number of respondents, not the number of people to whom questionnaires were sent or whose phone numbers were dialed. And also keep in mind that a low response rate turns any study essentially into a voluntary response study, which is of little value for inferring population values. It's almost always better to spend resources on increasing the response rate than on surveying a larger group. A full or nearly full response by a modest-size sample can yield useful results.

Surveys are not the only place where proportions pop up. Banks sample huge mailing lists to estimate what proportion of people will accept a credit card offer. Even pilot studies may mail offers to over 50,000 customers. Most don't respond; that doesn't make the sample smaller—they simply said "No thanks." Those who do respond want the card. To the bank, the response rate[7] is $\hat{p}$. With a typical success rate around 0.5%, the bank needs a very small margin of error—often as low as 0.1%—to make a sound business decision. That calls for a large sample, and the bank must take care in estimating the size needed. For our election poll calculation we used $p = 0.5$, both because it's safe and because we honestly believed p to be near 0.5. If the bank used 0.5, they'd get an absurd answer. Instead, they base their calculation on a proportion closer to the one they expect to find.

For Example SAMPLE SIZE REVISITED

A credit card company is about to send out a mailing to test the market for a new credit card. From that sample, they want to estimate the true proportion of people who will sign up for the card nationwide. A pilot study suggests that about 0.5% of the people receiving the offer will accept it.

QUESTION: To be within a tenth of a percentage point (0.001) of the true rate with 95% confidence, how big does the test mailing have to be?

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QUESTION: To be within a tenth of a percentage point (0.001) of the true rate with 95% confidence, how big does the test mailing have to be?

ANSWER: *Using the estimate $\hat{p} = 0.5\%$:*

$$ME = 0.001 = z^*\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96\sqrt{\frac{(0.005)(0.995)}{n}}$$

$$(0.001)^2 = 1.96^2\frac{(0.005)(0.995)}{n} \Rightarrow n = \frac{1.96^2(0.005)(0.995)}{(0.001)^2}$$

$$= 19{,}111.96 \text{ or } 19{,}112$$

That's a lot, but it's actually a reasonable size for a trial mailing such as this. Note, however, that if they had assumed 0.50 for the value of p, they would have found

$$ME = 0.001 = z^*\sqrt{\frac{pq}{n}} = 1.96\sqrt{\frac{(0.5)(0.5)}{n}}$$

$$(0.001)^2 = 1.96^2\frac{(0.5)(0.5)}{n} \Rightarrow n = \frac{1.96^2(0.5)(0.5)}{(0.001)^2} = 960{,}400.$$

Quite a different (and unreasonable) result.

. **Cars** What fraction of cars is made in Japan? The computer output below summarizes the results of a random sample of 50 autos. Explain carefully what it tells you.

z-Interval for proportion
With 90.00% confidence,
$0.29938661 < p(\text{japan}) < 0.46984416$

Confidence Intervals

We have been studying the materials which P (true p) was given.

like 13% of people in US are left-handed.

$\boxed{P = ?}$ ... In a real world, we usually don't know this value.

So last week, I introduced Coral Reef conditions.

what % of coral reef are infected with ~~a~~ the ~~particular~~ disease.
(aspergillosis)

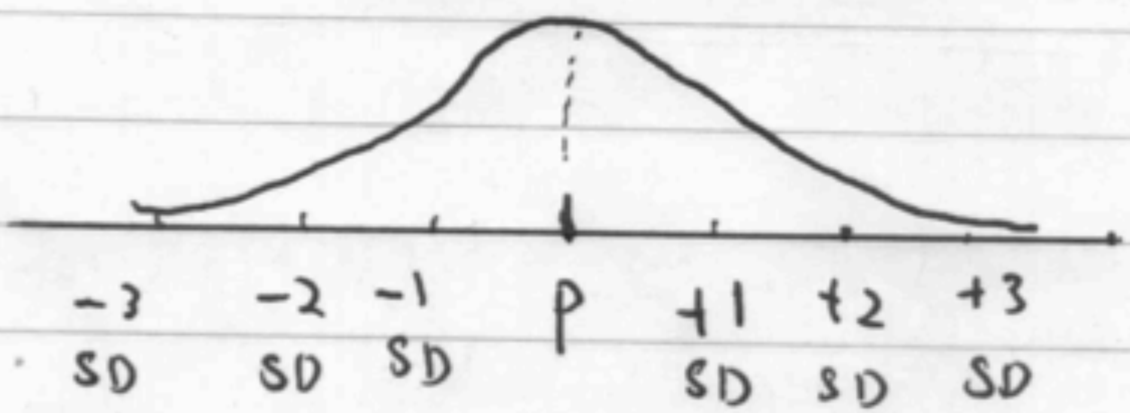
We have no idea P(true p) is.

Let's discuss what we know about sampling distribution.

We know sampling distribution represents many many samples.

true p in the middle.

(true average)

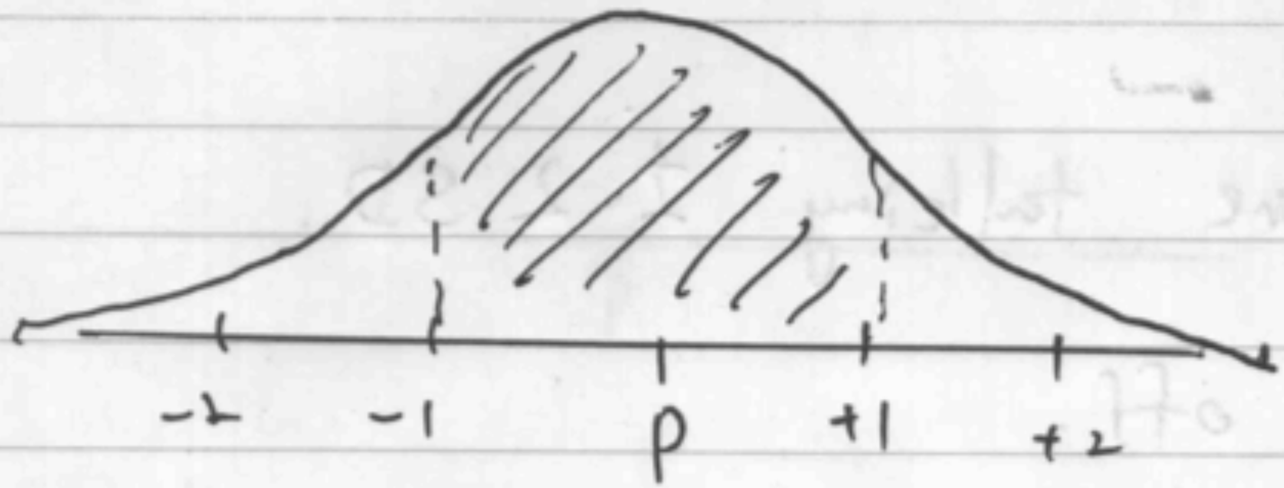


what we also learned

$$SD(\hat{p}) = \sqrt{\frac{pq}{n}}$$

sampling Distribution represents many many sample will look like.

If we have 100 people as sample.



68%

(68 ~~%~~ people out of 100 people) will be in this part of group.

we also know 95% or 95 people out of 100 people will be falls in ± 2 SD.

p

true p

This is what we assuming if we know the p.

The formula we used was. for 95%

$$p \pm 2 \cdot \sqrt{\frac{pq}{n}}$$

What if we don't know true p is?
So we have to do this little bit backwords.

95% ... we were talking ± 2 SD.

It's little bit off.

-2 $+2$

95%

z score : we were using ± 2 z score.

2.5% 95% 2.5%

tail %

Use TI 84

2nd vars 3 InvNorm

(.025)

2.5% 95% 2.5%

tail %

Use TI 84

2nd vars 3 InvNorm (.025)

$\pm 1.959 \approx \pm 1.96$

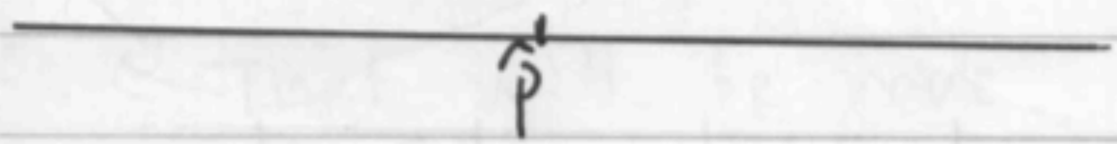
z^*

So it is $+1.96$ and -1.96 from now on.

We call this value ~~for~~ (critical value) and notate z^*.

How to find CI.

We have no idea how many coral reef are infected. So we took the sample. All we have is $\hat{p}$.
(pea hat)



$\hat{p}$ = sample proportion

n = Sample size

So we centered around $\hat{p}$.

Then we have to think ~~SD~~ standard deviation above and below.

$$SD(\hat{p}) = \sqrt{\frac{pq}{n}}$$

remember this is true p and true q. but we don't know it

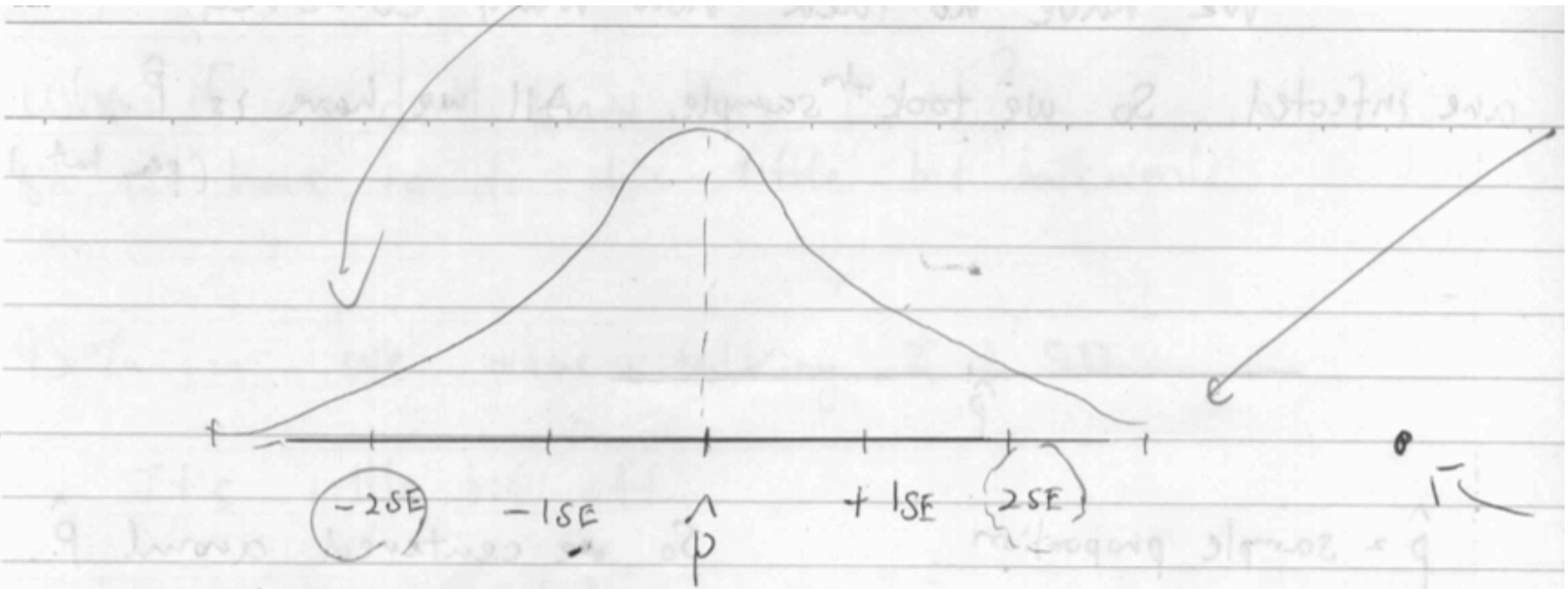
So we do is use the same formula

So we do is use the same formula

$$SE(\hat{p}) = \sqrt{\frac{\hat{p}\,\hat{q}}{n}}$$

We can't call this standard deviation. (It's not true p & q)

we call this standard error.



What we can learn from this, if my $\hat{p}$ is falling in the middle, I know the truth has to be near somewhere around there.

Go back to sampling distribution. 95% of Sample are going to be within 2 Standard deviation.

Think this is opposite way. I think that true p value has got to be probably within 2 standard deviation of my sample.

That how we build the confidence interval.
CI is trying to find the true proportion.

So what we do is we say

$\hat{p}$ in the middle we going add / subtract

$\hat{p} \pm 1.96 SE(\hat{p})$

↖ standard error of that proportion

→ fix this to $+1.96$
-1.96

No.

Date

95% of many many sample should create interval to catch the truth.
We don't know what true p is, and hope it is not out here. ⇐ That will be rare. I hope it lands somewhere in this range. $\hat{p} \pm 1.96 \cdot SE(\hat{p})$

To find CI (confidence Intervals) for true p.

Start with sample proportion $\hat{p} \pm 1.96 \cdot S.E(\hat{p})$

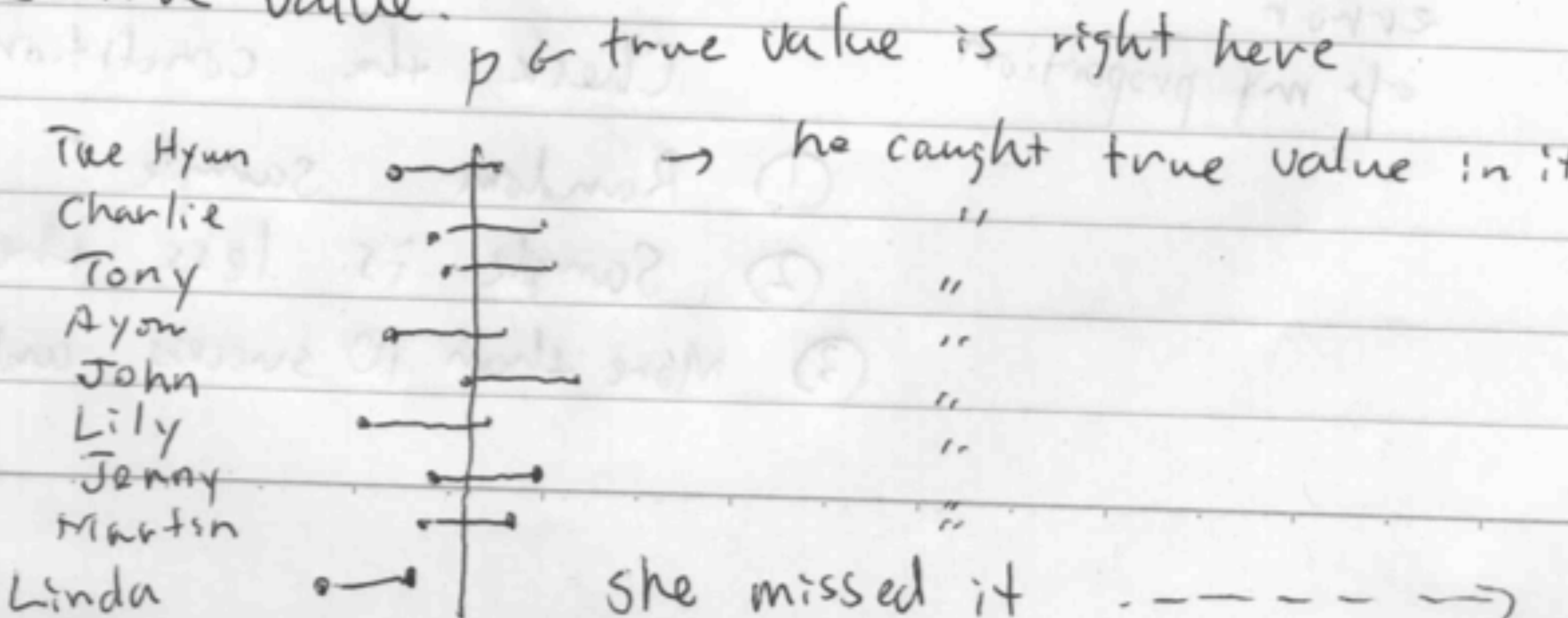
standard error of that proportion

This creates interval (low, high)

This is the interval where true p is.
We say 95% confident that true p lies in this interval.
What does 95% confident mean?

First of all it is not probability. It's NOT 95% chance.

what means if you run many many samples, you would have many many different $\hat{p}$s. 95% of those $\hat{p}$s creat intervals that contains the true value.



Her sample didn't contain the truth. She doesn't know that.
95% of these intervals are created by different samples would contain the true p.

Basically if we have 100 samples, 5 of them are not contain the true p. and 95 ~~%~~ of them would contain the true p.

Example (Go back 22 pages ~ 24 pages)

Sea Fans questions

$n = 104$ Coral reefs 54 infected

that means my $\hat{p} = \frac{54}{104} \Rightarrow 51.9\%$ $\quad 0.519$

(sample proportion)

This is not true p.

We can estimate here with the formula here.

$$SE(\hat{p}) = \sqrt{\frac{(0.519)(0.481)}{104}} = 0.049$$

opposite

standard error of my proportion

Check the conditions

① Random sample?

② Sample is less than 10% of population

③ More than 10 success and 10 failure

95% confident

$$\hat{p} \pm 1.96\, SE(\hat{p})$$

~~51.9~~ $\pm$ 1.96 (0.049)

.519

high

= .61504

low

30

. $42\cancel{296}$

.519 $\pm$ 0.09604

This back number has important name. It is called "Margin of Error"

(.4230 , .6150)

I'm 95% confident that the true proportion of infected coral reefs ~~are~~ with the disease is anywhere from 42.30% to ~~[illegible]~~ 61.50%.

Confidence Interval - part 2

$$\hat{p} \pm \underbrace{Z^* \cdot SE(\hat{p})}_{\text{Margin of error}}$$

Sample proportion
(what you saw in your sample)

Margin of error → it's not bad error
"sample is not going to be exact truth".

Review
Stadard Error like standard deviation

$$SE(\hat{p}) = \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

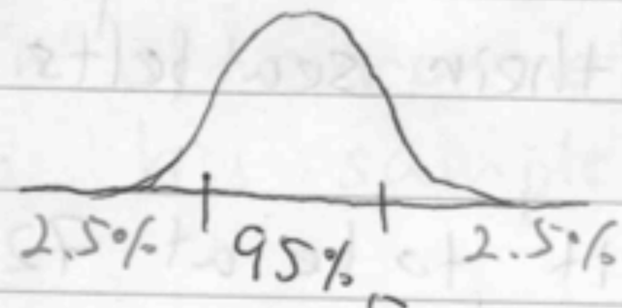
Z^* represent how confident you want to be.

Z^* represent how confident you want to be.

The typical one is 95%. We ~~usually~~ used to say ± 2 but ± 1.96. (little more acurate)

	how confident	tail percentage	Z^*	
Smaller interval	90%			less confident
	92%			
	95%	2.5%		
wider interval	99%			

For the last example we use $\pm$ 1.96 ~~for~~ as Z^* for 95% confidence Interval.



top tail

bottom tail

We want to be in between this Z star and this Z star

we call ~~those~~ this tail %.

This number we need to calculate Z^*.

$\boxed{\text{2nd}}$ $\boxed{\text{VARS}}$ #3 invert Norm invNorm(.025) Enter $1.9\overset{6}{\cancel{5}}49$

Let's do 90%

tail % $\frac{10\%}{2} = 5\%$

invNorm(.05) Enter ± 1.64

92%

tail % $\frac{8\%}{2} = 4\%$

invNorm(.04) ± 1.75

99% (0.005) $\frac{1\%}{2} = 0.5\%$ ± 2.58

invNorm(0.005)

The countries of Europe report that 46% of the labor force is female. The United Nations wonders if the percentage of females in the labor force is the same in the United States. Representatives from the United States Department of Labor plan to check a random sample of over 10,000 employment records on file to estimate a percentage of females in the United States labor force.

1. The representatives from the Department of Labor want to estimate a percentage of females in the United States labor force to within ±5%, with 90% confidence. How many employment records should they sample?

2. They actually select a random sample of 525 employment records, and find that 229 of the people are females. Create the confidence interval.

3. Interpret the confidence interval in this context.

4. Explain what 90% confidence means in this context.

5. Should the representatives from the Department of Labor conclude that the percentage of females in their labor force is lower than Europe's rate of 46%? Explain.